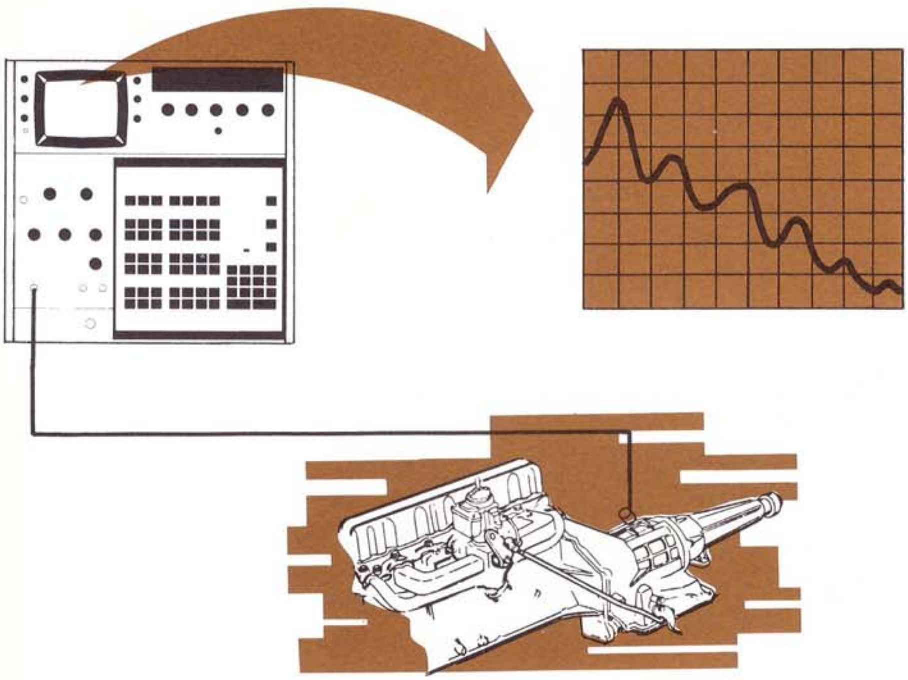
APPLICATION NOTE 140-4

Digital Auto-Power Spectrum Measurements

This application note is concerned with a single channel measurement of power using the HP Fourier Analyzer. The measurement is distributed in the frequency domain and is thus a power spectrum or (in some cases) a power spectral density. Several considerations in applying the Fourier Analyzer to the measurement must be exercised and are each discussed. These include classifying the type of data to be analyzed, choosing the proper window, calibrating the displayed spectra, normalizing broadband spectra to density measurements, and others. Statistical ensemble averaging and the energy measurement of transients are also treated.



HEWLETT PACKARD

I. INTRODUCTION

Spectral analysis of data has for a long time been popular in characterizing the operation of mechanical or electrical systems, etc. A type of spectral analysis, the power spectrum (and power spectral density), is especially popular because a "power" measurement in the frequency domain is one that engineers readily accept and apply in their solutions to problems. Single channel measurements (auto-power spectra) and two channel measurements (cross-power spectra) have both played important roles; however, the auto-power spectrum is perhaps the more popular. One reason is because it has been much easier to implement with analog techniques.

The measurement of the auto-power spectrum is but one of many measurements that may be performed on the HP Fourier Analyzer (e.g., cross-power spectrum, transfer function, coherence, auto- and cross-correlation, probability distribution function, etc.), but because the power spectrum enjoys considerable popularity, this note is devoted entirely to it. Consideration is given to periodic, random, and transient data; absolute signal calibration and accuracy; the analysis windows employed; signal averaging for statistical definition of signals and to recover signals in noise, and application-oriented examples to allow immediate use on practical problems.

Table 1 is a condensation of methods described in this note and should prove useful in making power measurements with the HP Fourier Analyzer.

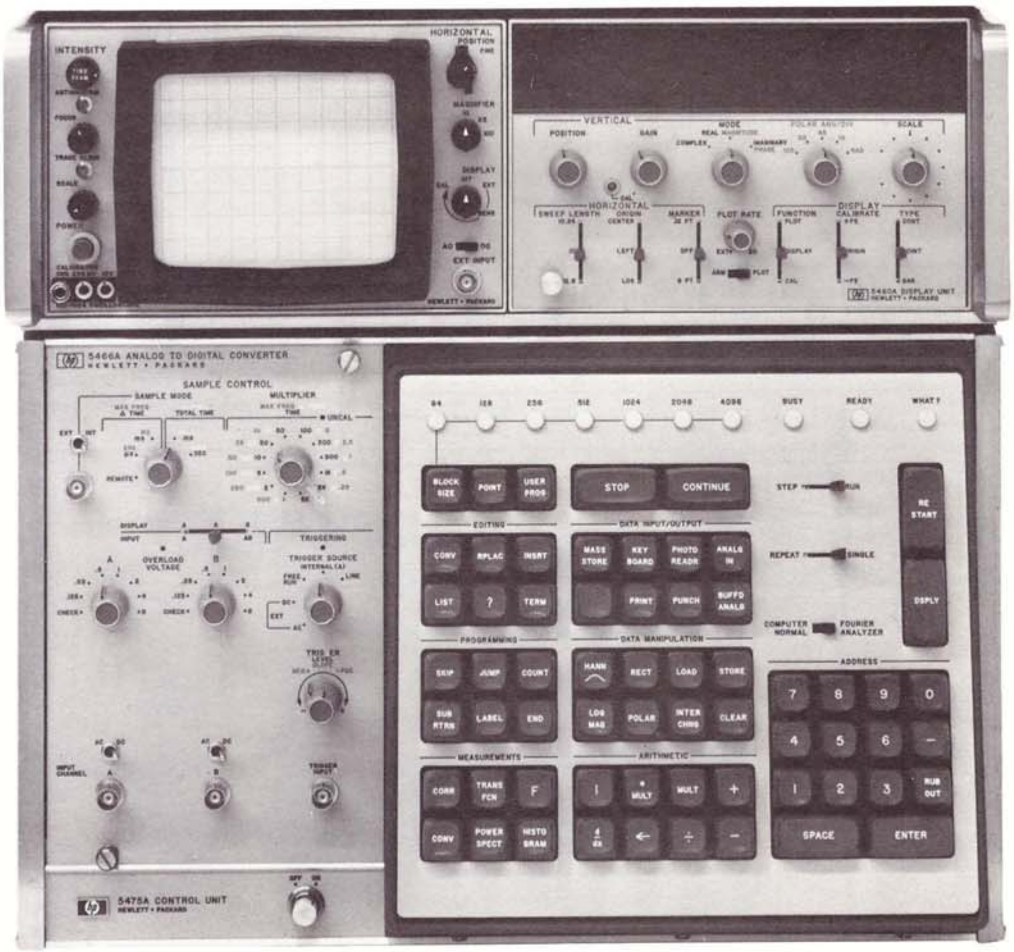
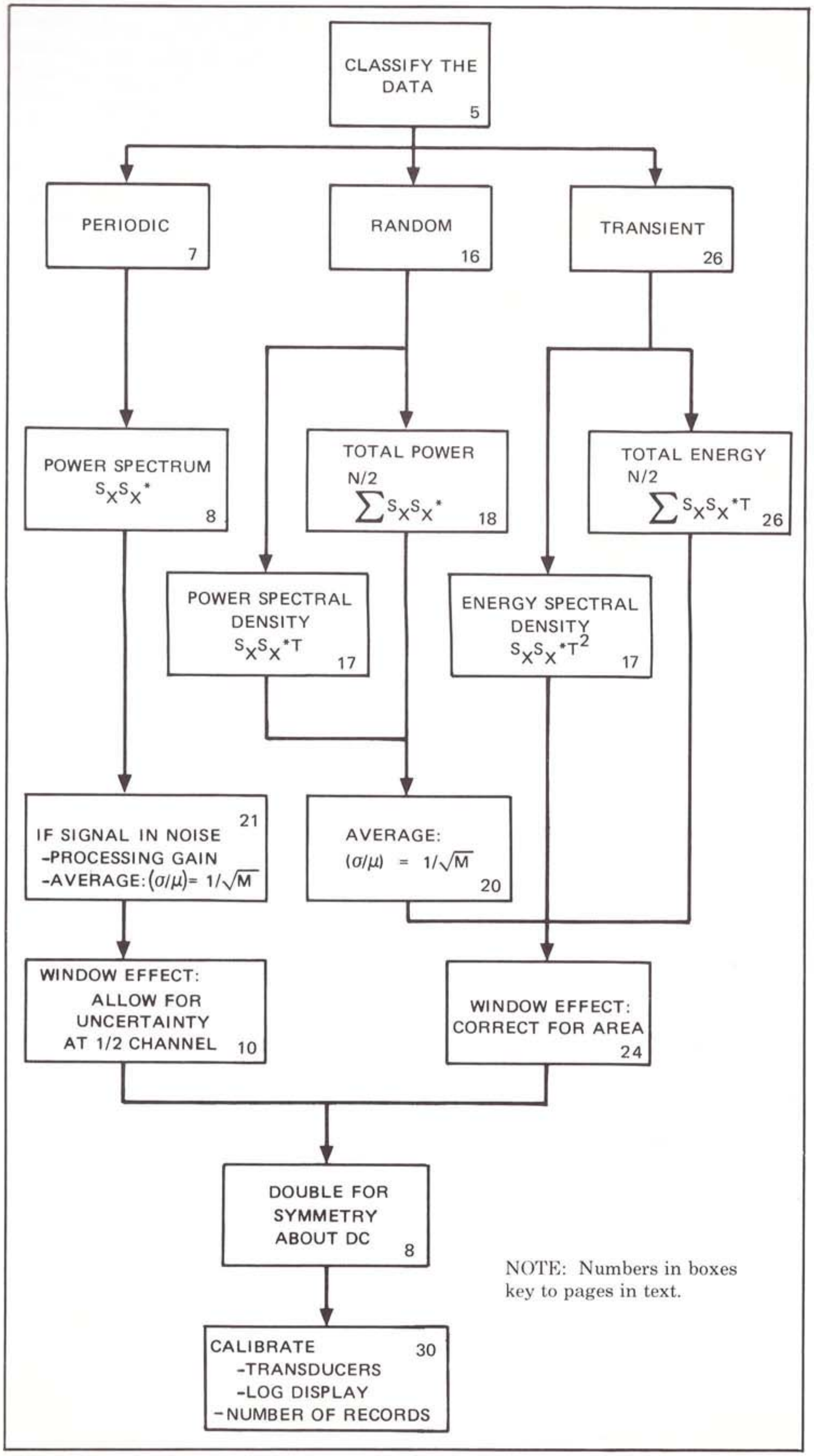


Table I. Measurement Method



NOTE: Numbers in boxes key to pages in text.

II. CLASSIFY THE DATA

A measurement of the acceleration of the case of a "noisy" rotating machine might produce data suspected of being random; but for the most part, it will be coherent, owing to the periodic qualities of the machine's operation. The time-domain "signature" of the machine is shown in Figure 1.

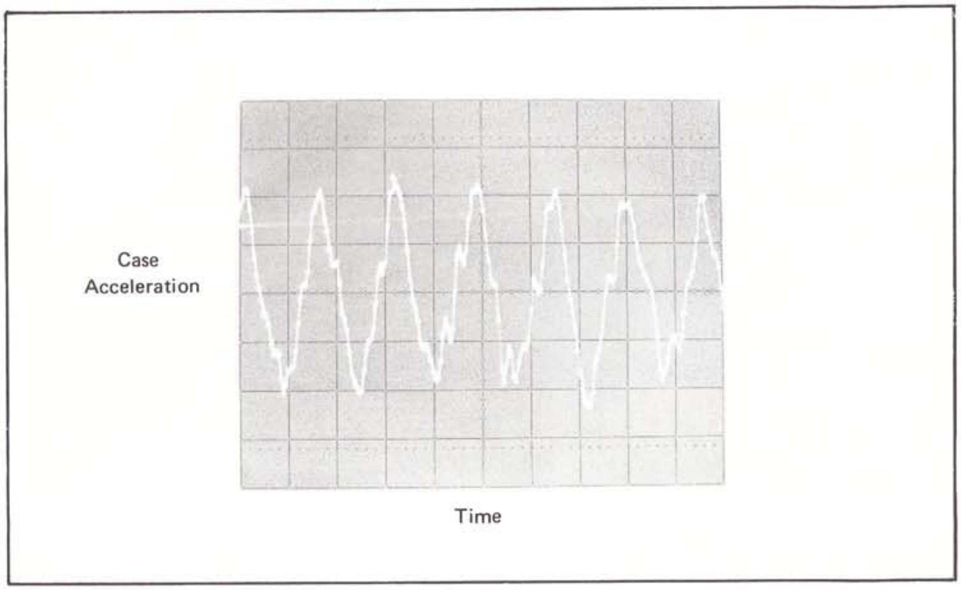


Figure 1

Superimposed on this coherent data though, may be random signals caused by a multitude of sources: random case accelerations from floor vibration caused by vehicle traffic, electrical noise in the measurement environment, a random disturbance in the machine itself, or other causes.

In addition to the analysis of coherent signals, random signals (Figure 2) frequently are characterized. Because different techniques are employed with random signals, it is important to recognize when a signal is random. Bridge vibrations caused by wind or traffic flow are random; the vibration of the component of a vehicle driven over a bumpy road is random; electronic noise in an amplifier is random.

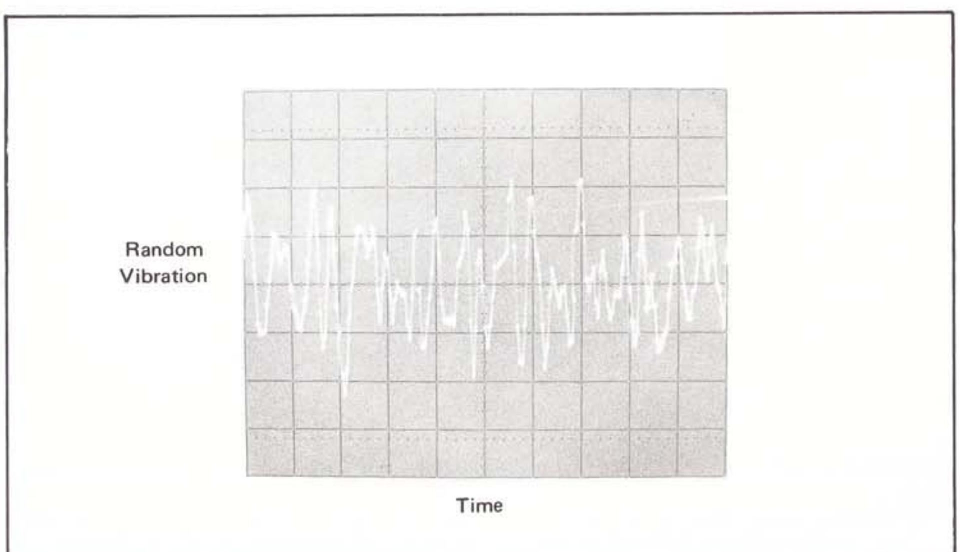


Figure 2

Another type of coherent signal requiring unique attention is a transient. Shown in Figure 3 is the decaying oscillatory response of a mechanical system, excited at a resonance by an impulse. The treatment of such signals is discussed later.

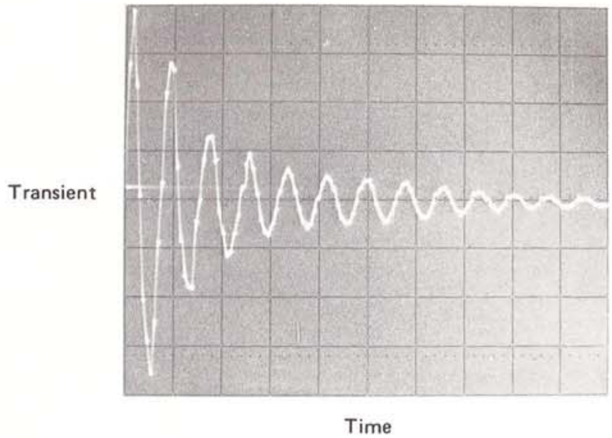


Figure 3

The expression of energy or power measurements is an important principle and is reviewed here.

A power measurement is considered to have the dimensions of the square of a signal. For instance, $voltage^2$ is synonymous with watts (across a one ohm resistor). In the same sense, $pressure^2$, $acceleration^2$, or $distance^2$, may be considered as expressions of power.

Power implies the flow of energy on a continuous basis and thus is a good description of a coherent or random signal. For a transient signal however, power has little meaning and it is best to talk in terms of the energy change during the period of the transient. Similar to the expression of power, energy is considered to have dimensions of $voltage^2$ seconds, $force^2$ seconds, etc.

The following sections are devoted to the measurement of each of the previously described data types. Section III deals with periodic data, Section IV with random data, and Section V with transient data. Signal averaging and windowing topics are introduced where applicable. Because of the continuity of the discussion and the interdependence of the sections, it is suggested that the text be read in order.

III. PERIODIC DATA (Periodic in the Window)

To illustrate amplitude scaling of spectra in the HP Fourier Analyzer, a periodic signal is used. Shown in Figure 4 is part of a 4-volt peak 10 Hz sine wave sampled for a 2-second record X(t):

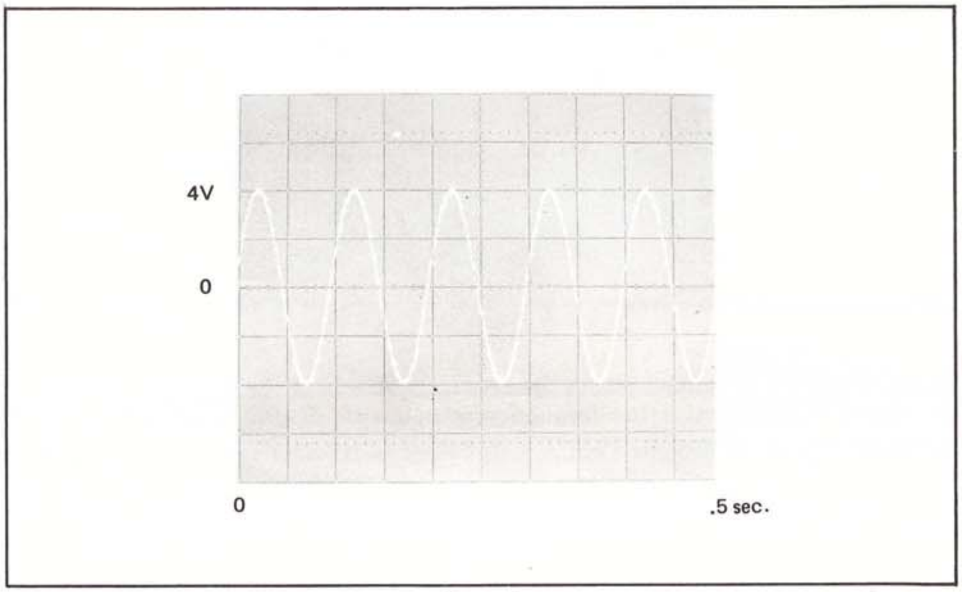


Figure 4

The Fourier transform S_x of this signal is shown in Figure 5A.

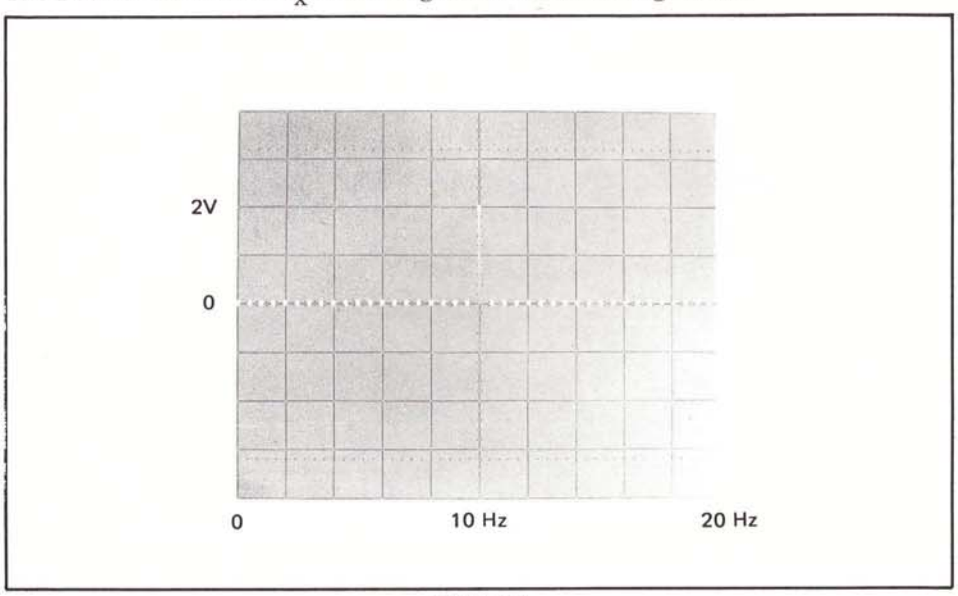


Figure 5A

Note that the magnitude of the Fourier transform is 2 volts peak instead of 4 volts peak as the sine wave sample would indicate. This is because the algorithm in the Fourier Analyzer is based on real time functions and has symmetry properties about DC; therefore it is sufficient to calculate and display one half of the spectrum, hence the 2-volt display.

Another example of this symmetry about DC appears in Figure 5B as:

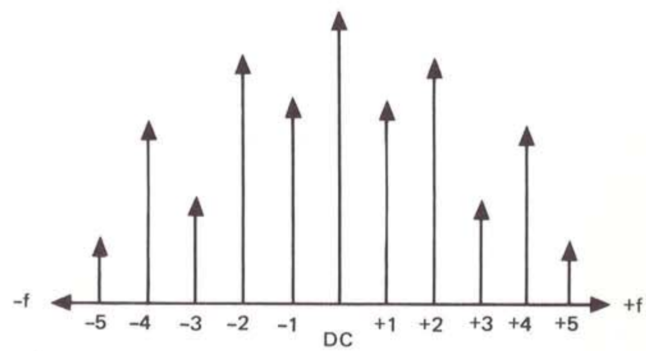


Figure 5B

Note that the DC component of the spectrum displays the total DC content of the signal whereas the other frequency components display only one half their respective spectral content.

One exception to this is the maximum frequency component F_{max} which, like the DC component, displays the total spectral content at this frequency.

The complex conjugate product of this voltage spectrum yields $S_x S_x^*$, the power spectrum (2 volts)^2 as illustrated in Figure 6.

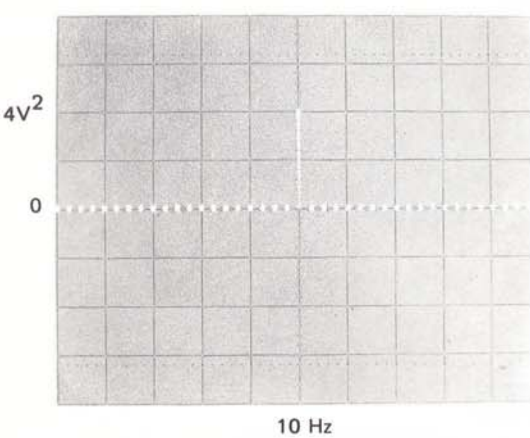


Figure 6

Note that the 4 volts^2 value represents half of the total (mean-square) power of the signal which is 8 volts^2.

In the next example, a calibrated 20 Hz sine wave is used as an input to the HP Fourier Analyzer, with its amplitude set to 0 dBm (50 ohm). The power 0 dBm is equivalent in this case to one milliwatt across a 50 ohm resistor, or 0.050 volt^2. On the HP Fourier Analyzer, the value displayed is thus half of 0.050 volt^2, 0.025 volt^2 or -16 dB:

$$-16 \text{ dB} = 10 \log_{10} .025$$

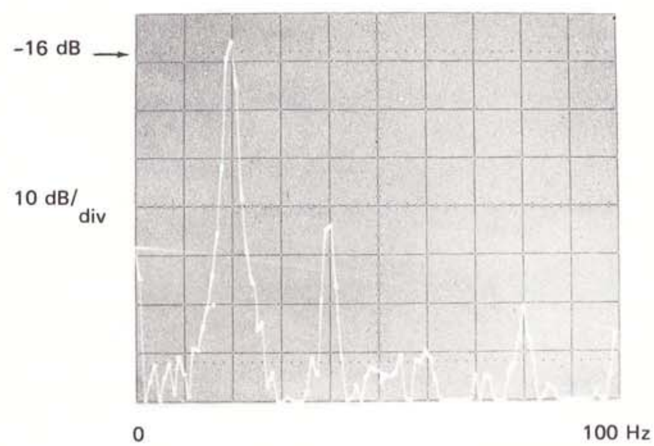


Figure 7

Muliplying the power calculated by 40 thus allows the display (Figure 8) to be calibrated in dBm (50 ohm):

$$0 \text{ dB} = 10 \log_{10} (40 \text{ x } .025)$$

Note in Figure 8 the second, third, and fourth harmonics at -38, -64, and -54 dBm, respectively.

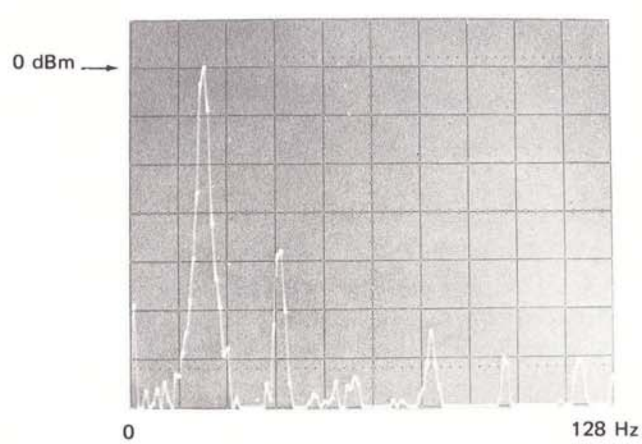


Figure 8

PERIODIC DATA (Non-Periodic in the Window)

Another example to consider is that of a 10-volt peak, 12.5 Hz sine wave sampled in a 1-second record (Figure 9):

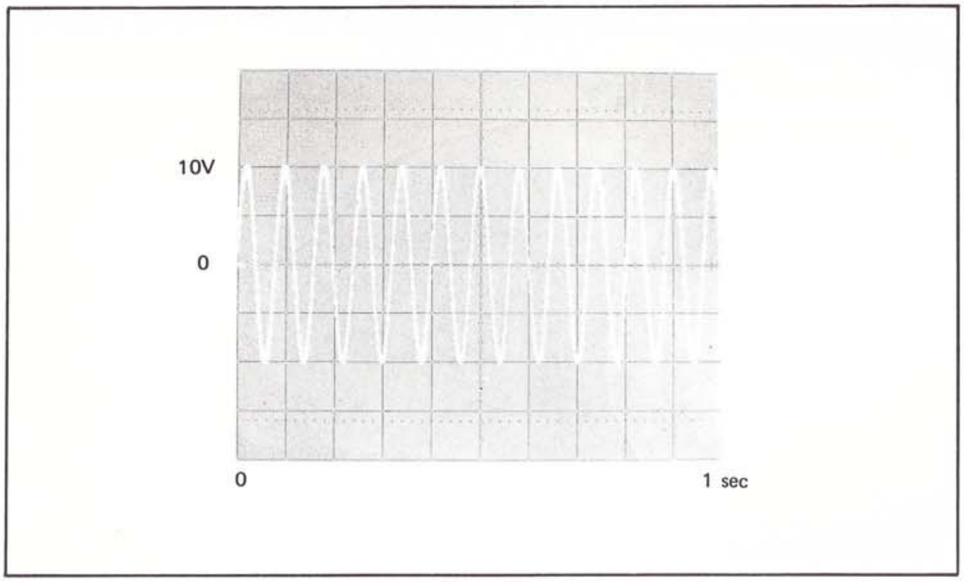


Figure 9

Expecting a 5-volt peak spectral line as the Fourier transform, what actually is obtained appears in Figure 10:

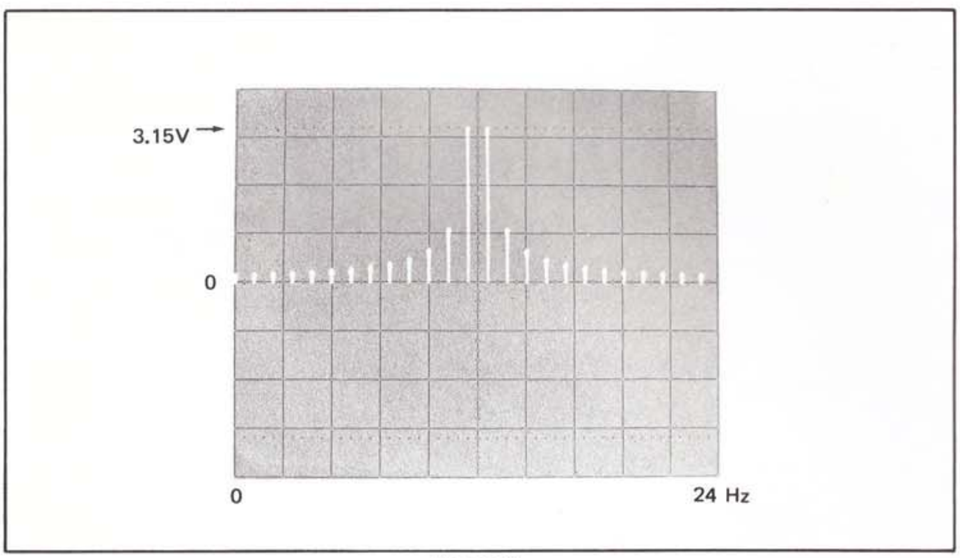


Figure 10

This is because the signal is non-periodic in the sample window and its frequency falls exactly between two discrete spectral lines and thus "leaks" out to adjacent lines. The log magnitude of this signal is shown in Figure 11.

The correct value should be 14 dB, representing 5 volts; however, the 10 dB displayed value is representative of the 3.15-volt result. Thus, there is a -4 dB error due to the measured frequency falling 1/2 of a frequency channel away from a discrete spectral line.

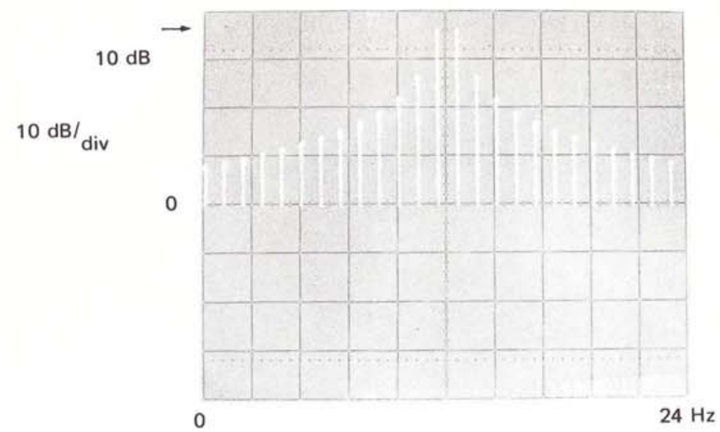


Figure 11

The nature of the "leakage" of the signal's power into adjacent frequency channels is characteristic of the rectangular sampling window employed. The Fourier transform of this window is shown in Figure 12 (positive half only).

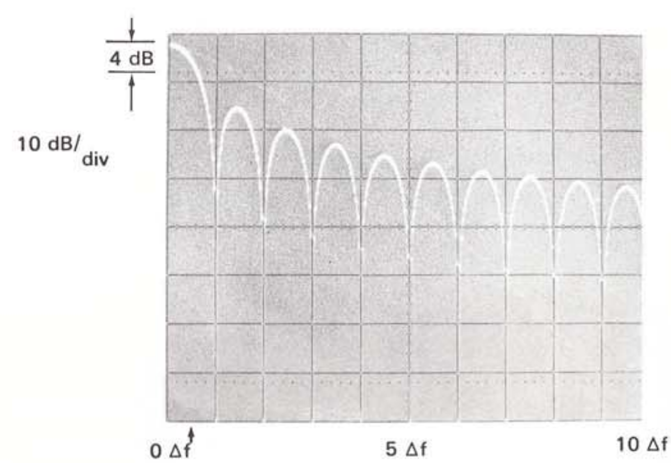


Figure 12

Note the -4 dB roll-off at 0.5 Δf from the center of the main lobe (at left edge of Figure 12).

Any number of windows may be employed, besides the rectangular window,[1] but a very convenient window to implement is the "hanning" window, callable by the HP Fourier Analyzer keyboard (Figure 13).

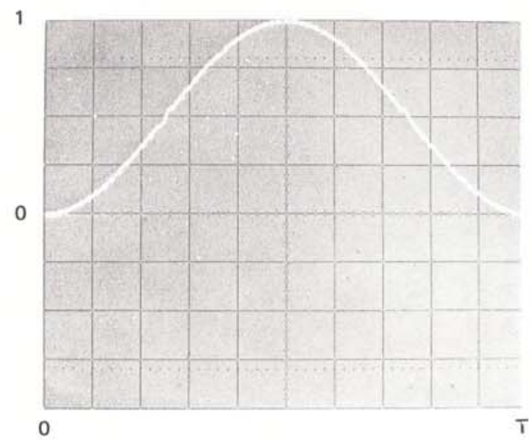


Figure 13

The Fourier transform of this window in Figure 14 indicates a faster roll-off of the side lobes at the expense of broadening the main lobe. The amplitude uncertainty is -1.4 dB at 0.5 Δf from the center of the main lobe:

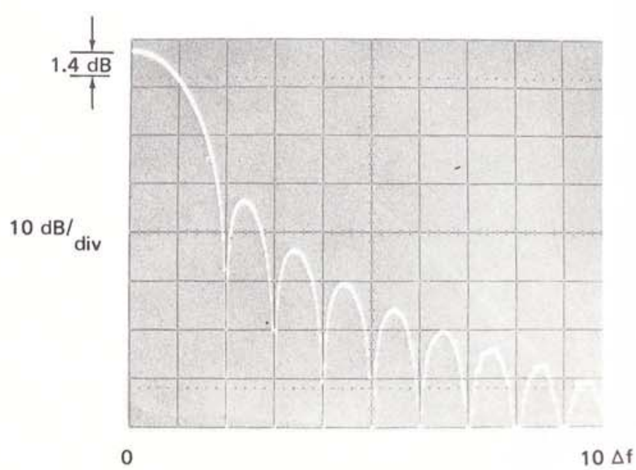


Figure 14

[1]See Reference 1.

The 12.5 Hz, 10-volt peak signal sampled through a "hanning" window appears as shown in Figure 15.

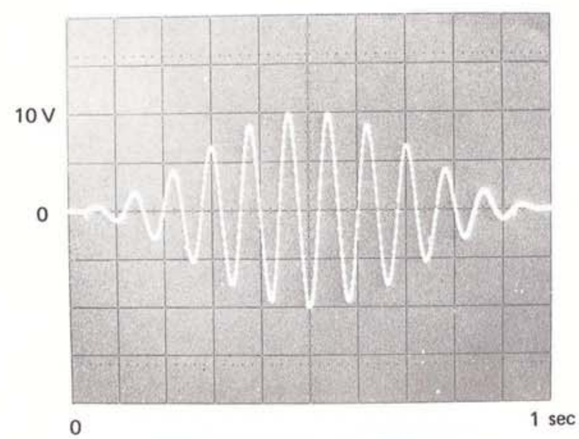


Figure 15

Other windows may easily[1] be implemented on the HP Fourier Analyzer. Shown below are windows [2] "P201" and "P301", each capable of producing amplitude accuracies of better than 99.9% (Figures 16-19).

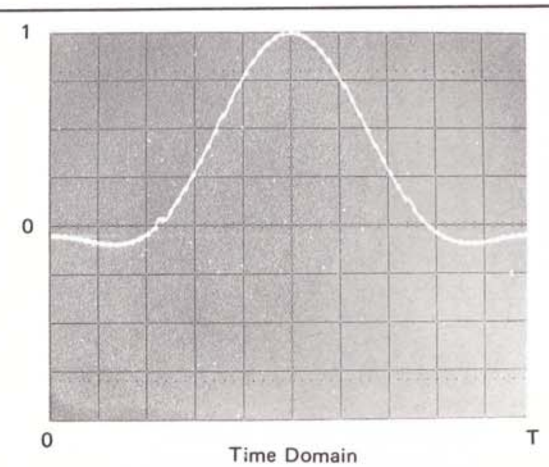


Figure 16

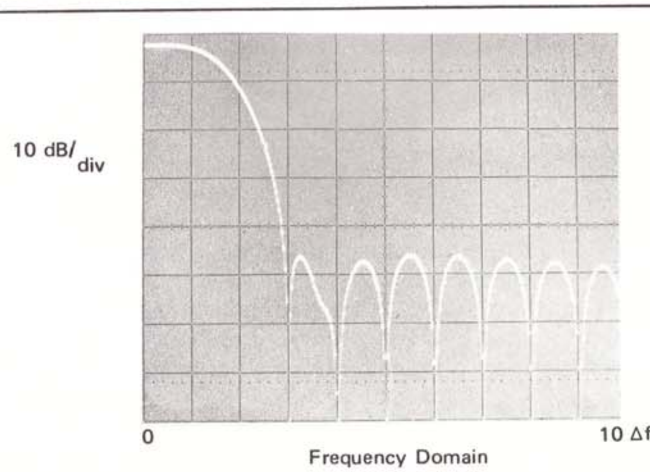


Figure 17

Window P201

[1]See Appendix I.

[2]See Reference 1.

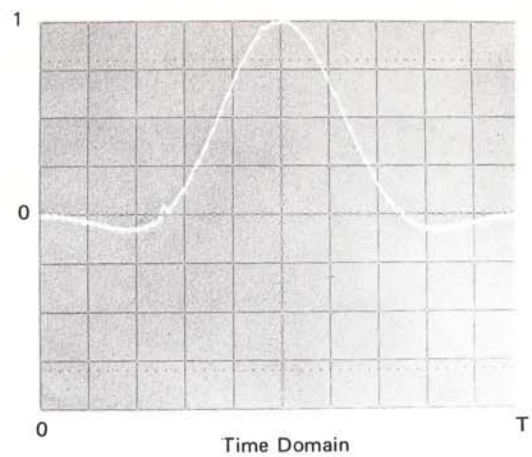


Figure 18

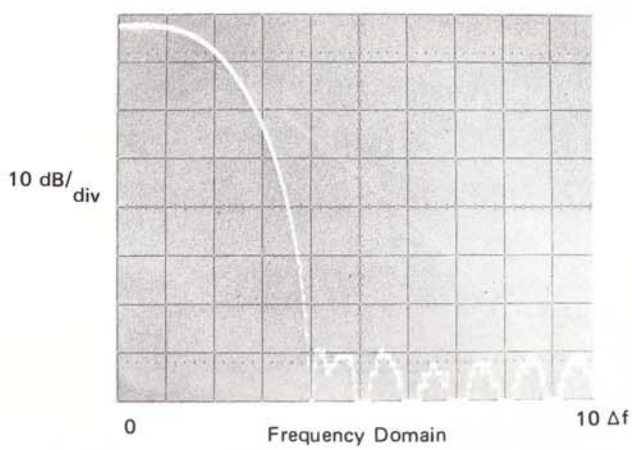


Figure 19
Window P301

The following example should reinforce these principles. It is desired to measure the power and frequency of a sine wave source. Knowing the approximate frequency as 30 Hz and allowing Δf to be 1 Hz, the measurement begins. The 1-second record is taken, multiplied by window P301, and Fourier transformed. The complex conjugate product yields in log magnitude what is shown in Figure 20.

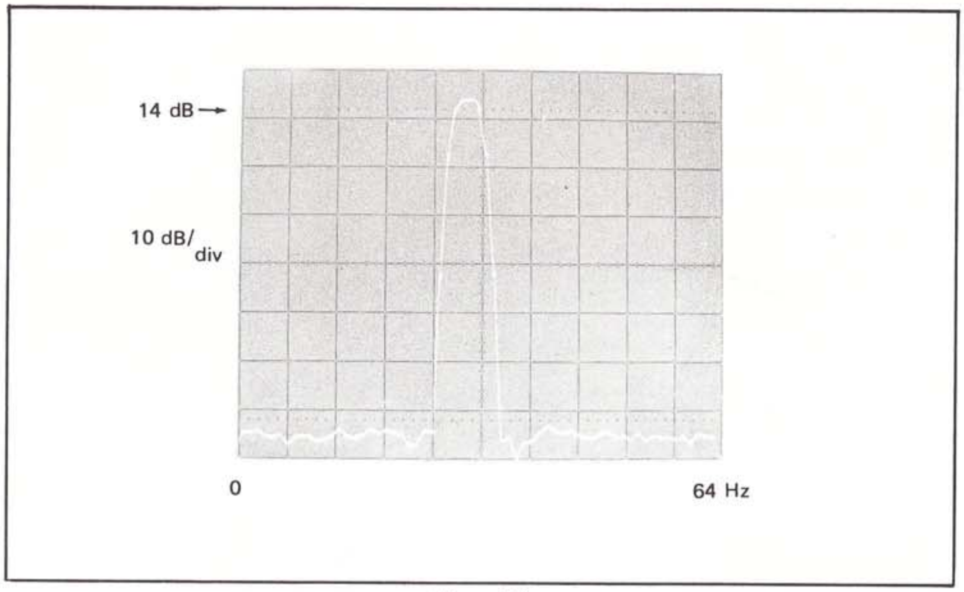


Figure 20

The fact that both the 30 and 31 Hz lines are nearly equal magnitude reveals that the frequency is close to 30.5 Hz. The amplitude value of 14 dB indicates a 25 V^2 power. Doubling this value for true power gives 50 V^2 or 50 watts (across a 1 ohm resistor).

If the source of this sine wave had been the acceleration of a shake table measured with an accelerometer calibrated at 2 volts per g (32.17 ft/sec^2), the power would be expressed as:

$$50\ V^2 \bullet \frac{g^2}{4\ V^2} = 12.5\ g^2$$

IV. RANDOM DATA

In a previous example, a 4-volt peak 10 Hz sine wave was analyzed with 0.5 Hz Δf resolution. If the measurement is duplicated with 1 Hz Δf resolution, the result will be that illustrated in Figure 21.

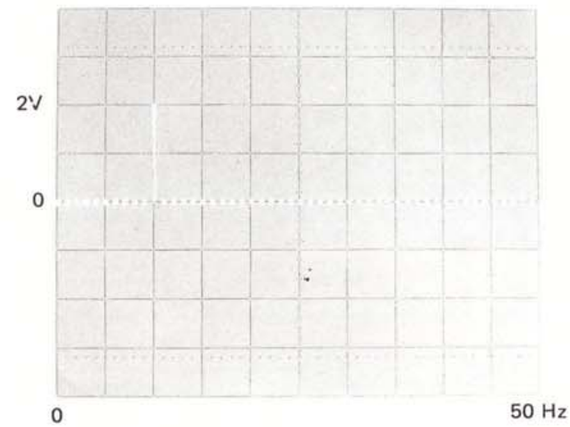


Figure 21

The same answer is obtained regardless of the observation bandwidth of the Fourier transform. The coherent signal's power spectrum remains the same.

Such is not the case when a wideband signal is measured. In the following example, a power spectrum ensemble average of 100 samples[1] of a random noise source, band-limited at 150 Hz is displayed in Figure 22.

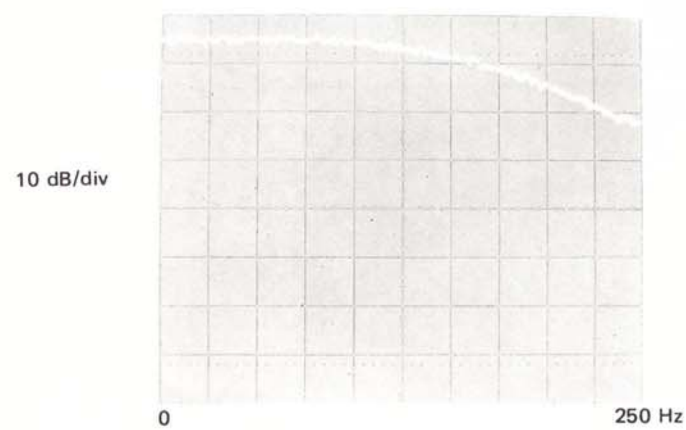


Figure 22

The frequency resolution is 1 Hz. Now, the measurement is duplicated with a 2 Hz Δf resolution and shown in Figure 23.

[1]See Appendix II.

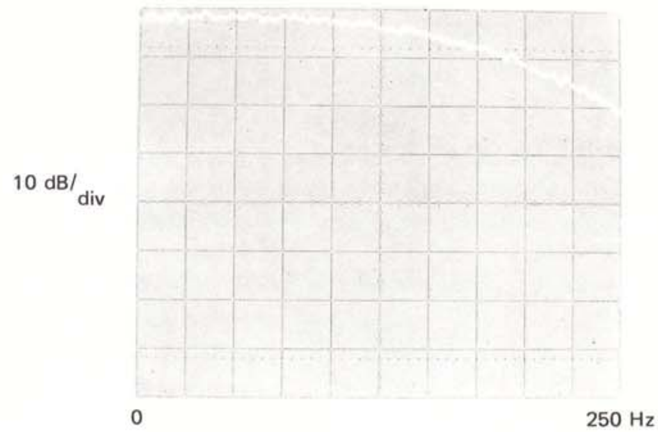


Figure 23

Note that the magnitude is twice as large (3 dB). The same measurement performed with various bandwidths of resolution would reveal that the magnitude varies proportional to the bandwidth used. Thus, it is necessary to normalize the measurement by dividing by the bandwidth to obtain invariant results on the same data. These normalized measurements are power spectrum density measurements (PSD).

Specifically on the HP Fourier Analyzer, the Fourier transform of a time signal X(t) is displayed in frequency "channels" Δf apart. Thus, the voltage distributed in the frequency domain is:

$$S_X \quad \frac{\text{volts}}{\text{channel}} \quad , \quad \text{where } S_X = F \quad X(t) \quad .$$

By applying the units conversion factor

$$\frac{1}{\Delta f} \quad \frac{\text{channels}}{\text{Hz}} = T \quad \frac{\text{channels}}{\text{Hz}} ;$$

$$S_X \quad \frac{\text{volts}}{\text{channel}} \bullet T \quad \frac{\text{channels}}{\text{Hz}} = S_X T \quad \frac{\text{volts}}{\text{Hz}}$$

The complex conjugate product yields

$$S_X S_X{}^* T^2 \quad \frac{\text{volts}^2}{\text{Hz}^2} \text{ or } \frac{\text{volts}^2 \text{ seconds}}{\text{Hz}}$$

which is the energy spectral density. Dividing by the period T yields the power spectral density (PSD):

$$\frac{S_X S_X{}^* T^2}{T} = S_X S_X{}^* T \quad \frac{\text{volts}^2}{\text{Hz}} \quad .$$

The total power in the band is simply:

$$\int_{DC}^{F_{max}} S_x S_x{}^* T \, df \to \Delta f \sum^{N/2} S_x S_x{}^* T = \sum^{N/2} S_x S_x{}^*.$$

As stated before in the sinusoidal case, half the power is displayed so the observed value must be doubled (except for the DC and F_{max} components which already display true values) to obtain the true power spectral density.

This can be demonstrated by measuring the power spectral density of a calibrated noise generator.

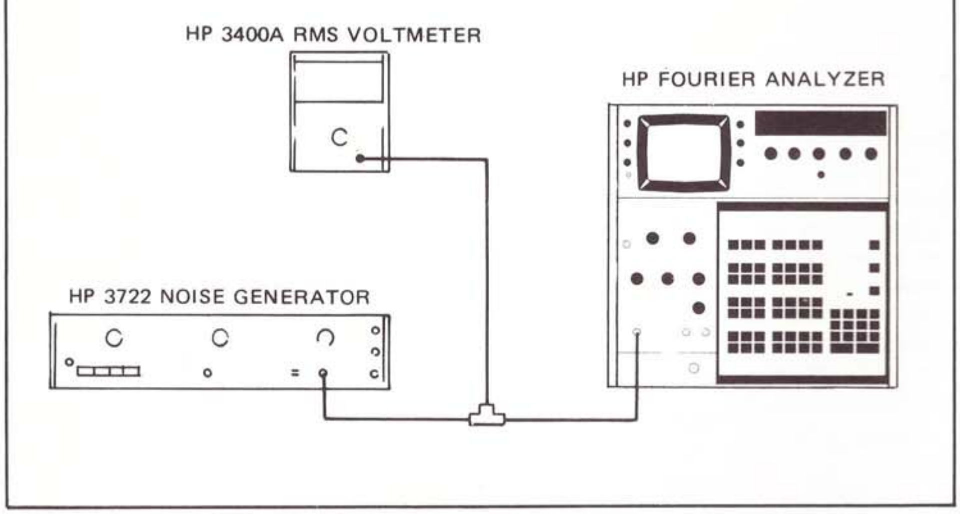


Figure 24

The output voltage of the noise generator is set to 1 Vrms, or a total power of 1-volt^2 (mean-square) and double-checked on a true-rms voltmeter, with an accuracy of ±1%.

The HP Fourier Analyzer corroborates this result (see Figure 25) with an ensemble average[1] of 400 samples of a noise generator output, band-limited at 500 Hz (the cutoff frequency).

[1]See Appendix III.

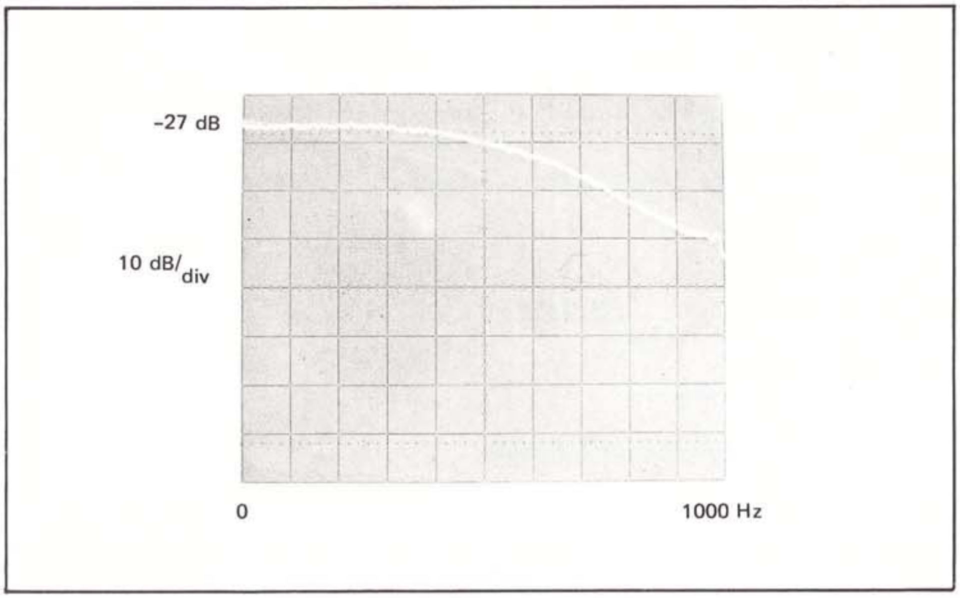


Figure 25

The PSD measured at the low end of the band was:

$$.00194\ V^2/Hz$$

which is in agreement with the expected value of approximately:

$$\frac{1\ V^2}{500\ Hz} = .002\ \frac{V^2}{Hz}$$

The power spectral density summed over all positive frequencies yields:

$$\int_{DC}^{F_{max}} S_X S_X{}^*T\ df \rightarrow \Delta f \sum^{N/2} S_X S_X{}^*T = \sum^{N/2} S_X S_X{}^*\ \text{volts}^2,$$

half the total power.

Summing the power in all frequency channels and doubling yields:

$$.992\ V^2$$

as the total power[1] measured, which is within the accuracy of the rms voltmeter.

[1]See Appendix IV

AVERAGING

The technique of averaging is useful in two areas: (1) when the signal itself is random and must be described statistically, and (2) when there is a deterministic signal in random noise to be detected. Averaging random signal power spectra yields an approximately normal distribution about the mean value of the power level according to the Central Limit Theorem.[1] For many averages M, it can be shown that:

$$\left(\frac{\sigma}{\mu}\right)_{\text{average}} = \frac{1}{\sqrt{M}}$$

Where M = number of averages

μ = mean signal power

σ = one standard deviation from μ

In the following example, averaged power spectra measured from a white noise generator set to 50 Hz bandwidth. Ensemble averages of 10 and 100 samples are shown in Figures 26 and 27, respectively.

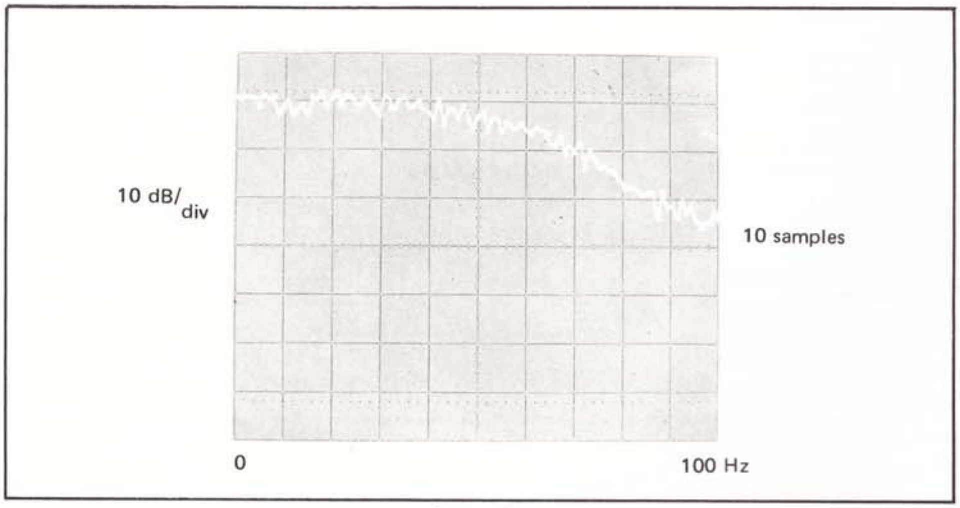


Figure 26

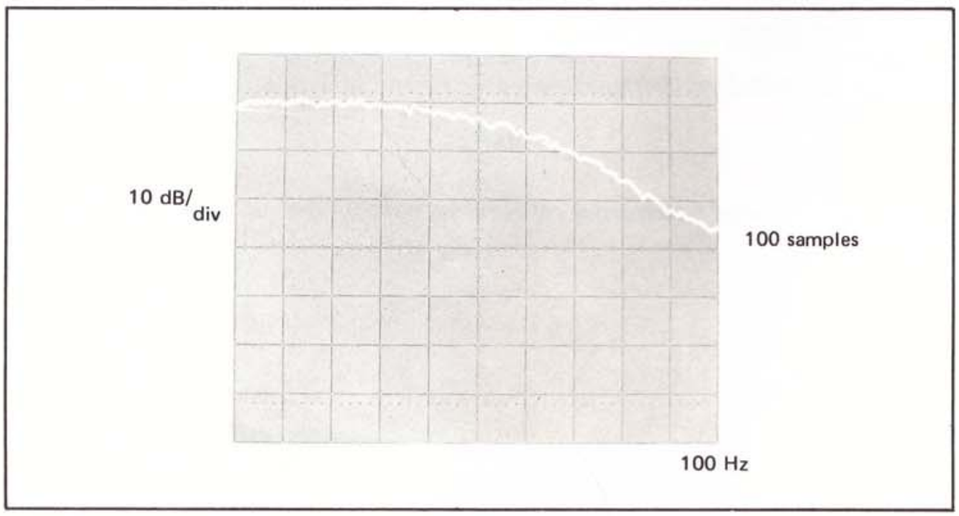


Figure 27

[1]See Reference 3

Note that the variance of the data decreases with more averages. Figure 28 depicts this variance.

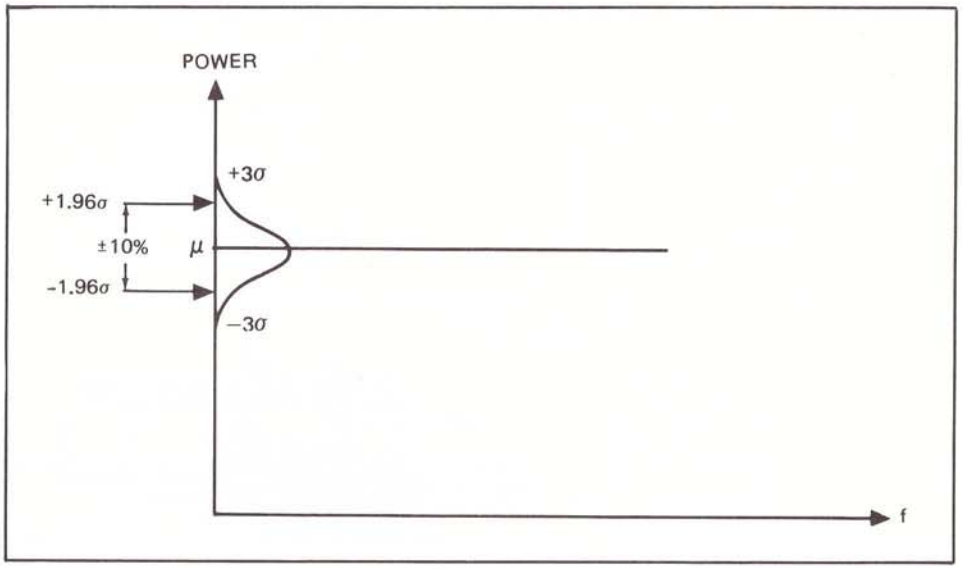


Figure 28

With 99.7% confidence, the true mean will fall within $\pm 3\sigma$ around the displayed averaged mean. A confidence of 95% implies $\pm 1.96\sigma$, so based on the objective of computing the true mean with no more than ±10% inaccuracy with a 95% confidence:

$$10\% = .10 = \frac{1.96\sigma}{\mu} = \frac{1.96}{\sqrt{M}}, \quad \text{or}$$

$$M = 384 \text{ averages necessary.}$$

For a coherent signal in random noise, the following example (Figure 29) demonstrates the technique. The wideband signal to noise ratio is measured to be -17 dB for a coherent signal in white noise band-limited at 50 Hz. Using a frequency resolution of 1 Hz, it may be assumed that the noise power falls in the first 50 channels of the data block (in the frequency domain).

For simplicity, assume the coherent signal is periodic in the window and its spectrum occurs in one frequency channel. Now, only 1/50 of the noise interfers with the signal's detection, representing a 17 dB improvement as a result of processing gain.

Also, the signal-to-noise ratio now (narrow band) is 0 dB, or the signal and noise power are equal.

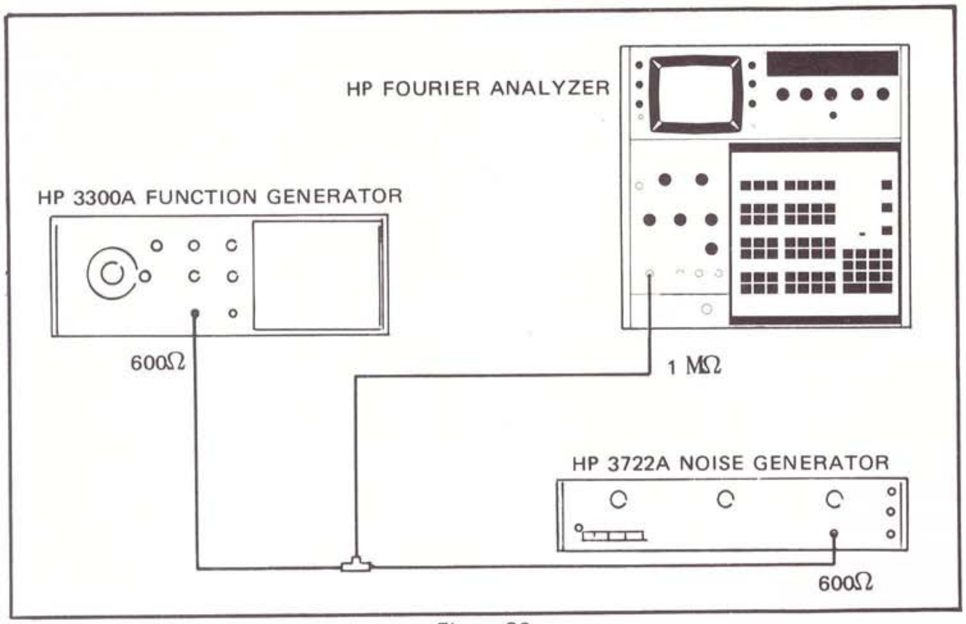


Figure 29

Shown (Figures 30-33) are the signal alone and with noise. (A 0.1-volt peak sine wave is measured in a 0.5-volt rms noise level. Half of the power is displayed in each case):

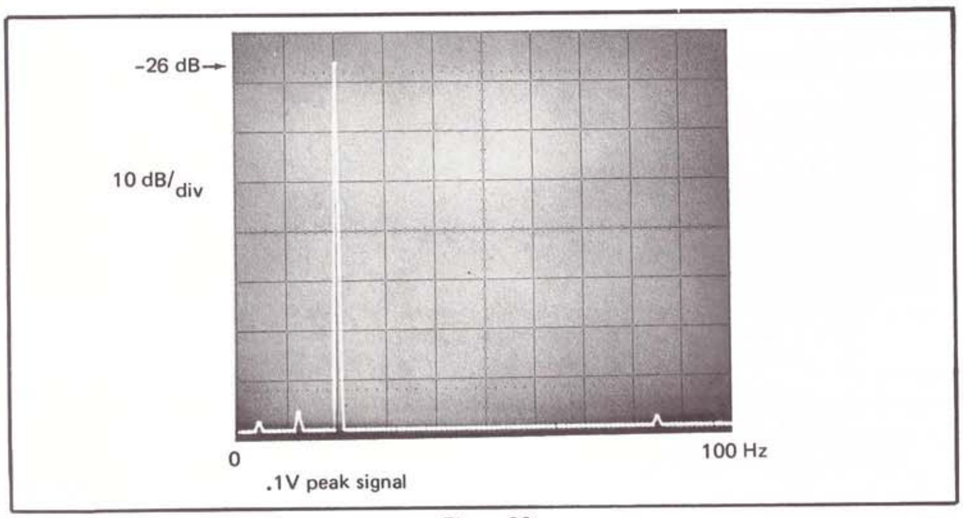


Figure 30

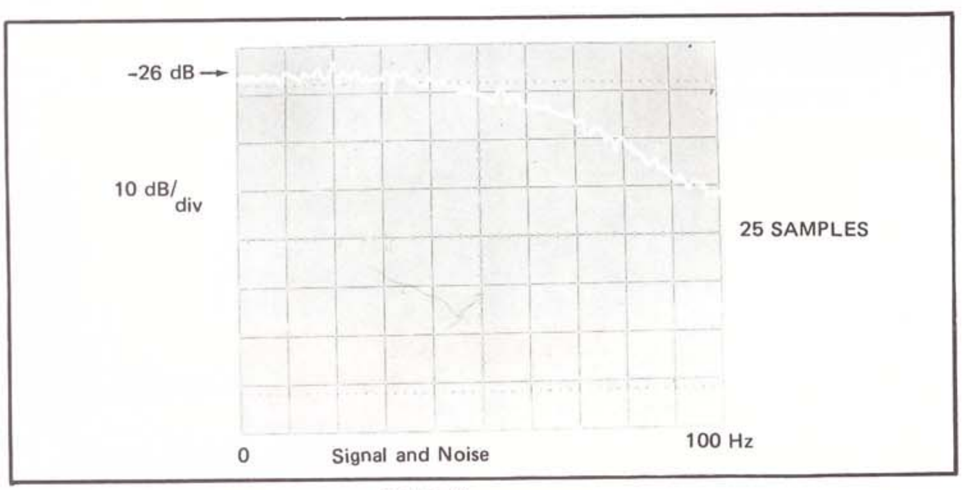


Figure 31

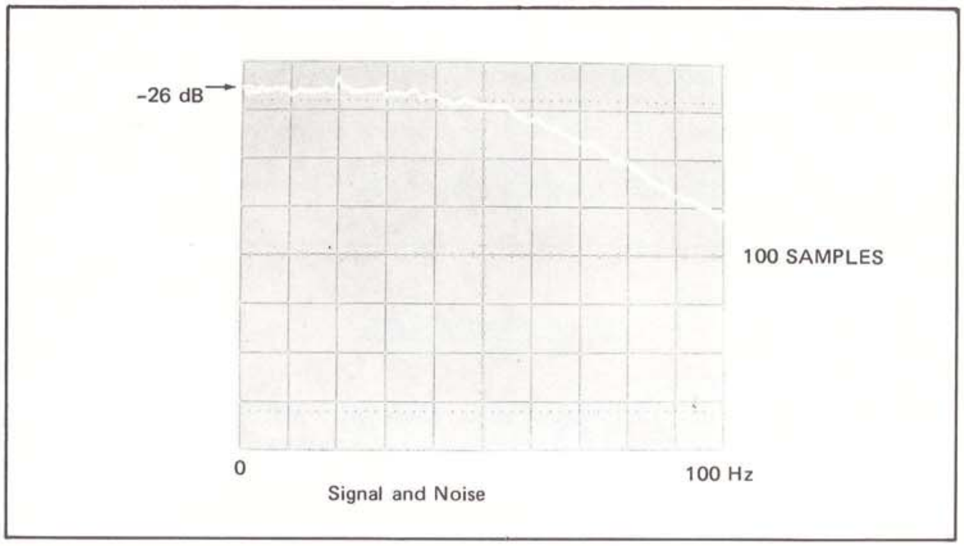


Figure 32

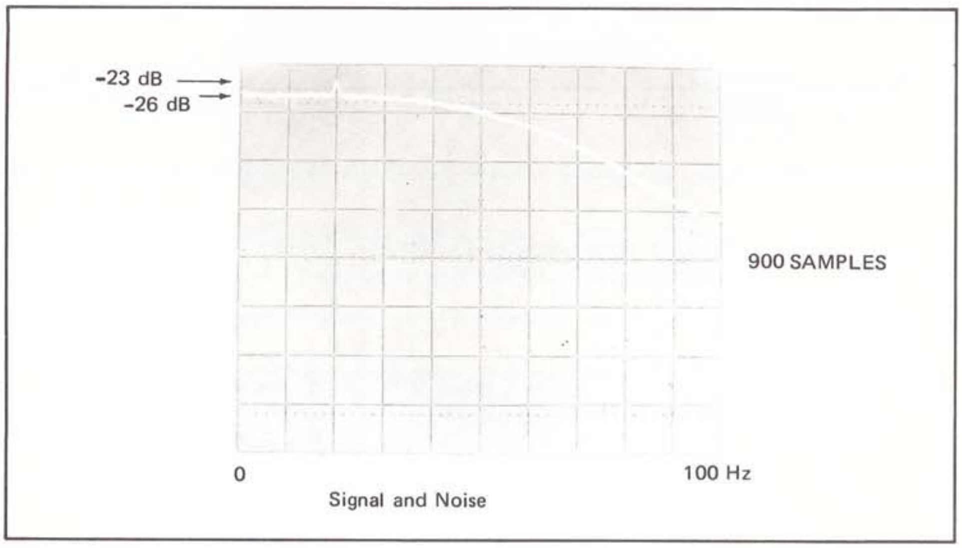


Figure 33

Note that the signal appears as 3 dB above the noise level, because the total power is double at that frequency. The reduction of the noise variance to make the signal discernible follows the same statistical basis as described on pages 20 and 21.

WINDOW EFFECT

The effect of the window in the case of a random signal can be conveniently viewed as a filter about each spectral line. This filter's effect can be expressed in terms of the area under the square of the filter's shape (or lineshape). This is because the noise (or wideband signal) power passed is proportional to the area under the square of the filter lineshape (Figure 34).

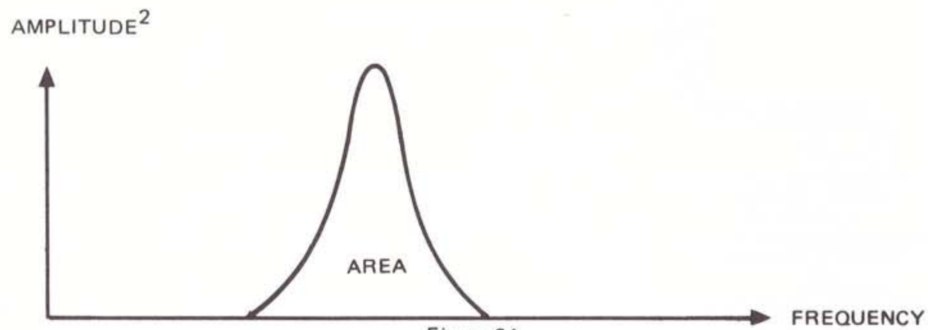


Figure 34

The rectangular sampling window used in the HP Fourier Analyzer (Figure 35) has an area under the square of the lineshape equal to 1.

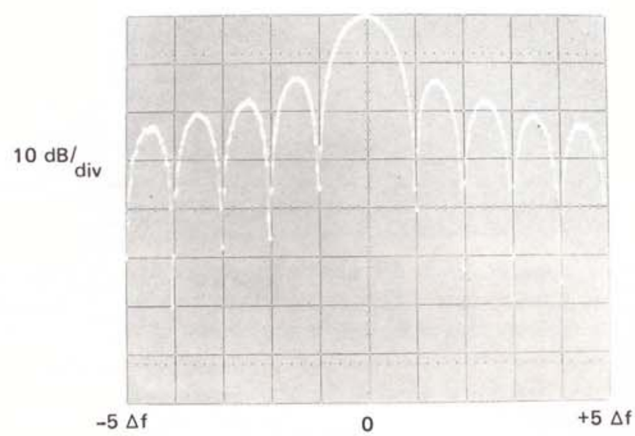


Figure 35

In the case of a hann window, the area under the square of the lineshape is 3/8.[1]

Correction for a window is made as follows:

$$\text{True PSD} = \frac{\text{Measured PSD}}{\text{area under window lineshape}^2}$$

[1]See references 1 and 2 for various window characteristics

MEAN-SQUARE Versus ROOT-MEAN-SQUARE

Spectral densities measured using analog techniques frequently are expressed as:

$$\frac{\text{volts}_{\text{rms}}}{\sqrt{\text{Hz}}}$$

rather than

$$\frac{\text{volts}^2\ \text{(mean-square)}}{\text{Hz}}$$

These factors must be considered when expressing comparable digital and analog results.

V. TRANSIENT DATA

Because signal transients last only a finite length of time, their expression in terms of total energy or energy density is more meaningful than a power concept. As stated before, energy spectral density is:

$$S_X S_X^* T^2 \qquad \frac{\text{volts}^2\ \text{seconds}}{\text{Hz}}$$

Summing all these spectral components from DC to F_{max} and doubling gives the approximate total energy:

$$2 \int_{DC}^{F_{max}} S_X S_X^* T^2 \, df \rightarrow 2\, \Delta f \sum^{N/2} S_X S_X^* T^2$$

The same total energy in the time domain is obtained by summing the squares of each voltage sample over the record T:

$$\int_{o}^{T} X^2 \, dt \rightarrow \Delta t \sum^{N} X^2,$$

by Parseval's Theorem:

$$\Delta t \sum^{N} X^2 = 2\, \Delta f \bullet T^2 \sum^{N/2} S_X S_X^*$$

$$\frac{\Delta t}{T} \sum^{N} X^2 = 2\, \Delta f \bullet T \sum^{N/2} S_X S_X^*$$

$$\frac{1}{N} \sum^{N} X^2 = 2 \sum^{N/2} S_X S_X^*$$

$$\text{since} \quad \frac{\Delta t}{T} = \frac{1}{N} \quad \text{and} \quad T = \frac{1}{\Delta f}.$$

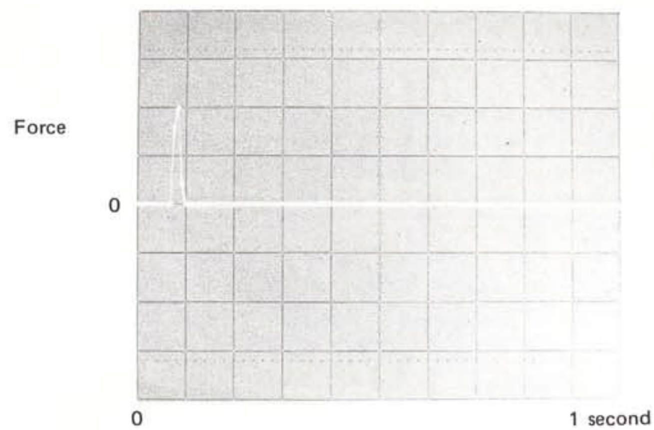


Figure 36

An example of this is the measurement of a force impulse caused by a hammer blow. In the time domain, the force is illustrated in Figure 36.

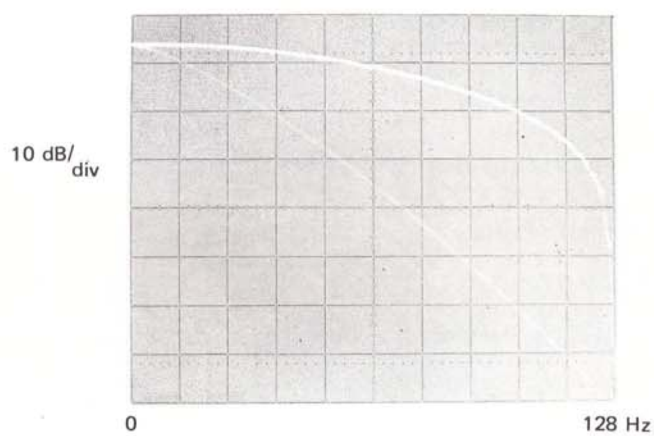


Figure 37

In the frequency domain, the power spectrum appears as seen in Figure 37.

Summing over the frequency channels (not including DC and F_{max}), doubling for total energy and adding the DC and F_{max} components gives:

$$.0026 \text{ volts}^2 \text{ seconds}$$

The results obtained by squaring the time signal and summing over the record interval is:

$$.0026 \text{ volts}^2 \text{ seconds,}$$

which demonstrates Parseval's Theorem.

A conversion factory of 20 lb per volt from the force cell calibration yeilds an energy value of:

$$(.0026 \text{ volt}^2 \text{ seconds}) \left(\frac{400 \text{ lb}^2}{\text{volt}^2} \right) = 1.04 \text{ lb}^2 \text{ seconds}$$

VI CONCLUSION

As a conclusion to the discussion, an example measurement is made to tie in each of the previous topics.

It is desired to determine the vertical displacement PSD of an automotive component driven over a road course. A tape recording of the signal from an accelerometer mounted on the component provides the data, with a record length of 30 minutes.

For convenience, Table I is repeated here to emphasize the method of data reduction. First, this data is classified as random, because the road is bumpy and known to excite the vehicle in a random fashion. A power spectral density measurment is required, averaged to produce statistical certainty.

A frequency resolution of 0.5 Hz out to a frequency of 100 Hz is required, so a data blocksize of 1024 points (Δf = 0.5 Hz and F_{max} = 256 Hz) and a 48 dB/octave anti-aliasing filter set at 100 Hz is adequate to insure an 80 dB dynamic range measurement.[1]

The sampling period T for each data record is 2 seconds, so ensemble averaging 400 records will insure good statistical certainty of a representative sample.

Amplitude accuracy of within one decibel is desireable so the data is "hanned" twice.[2]

Using the keyboard program in Appendix V yields a calibrated log display where:

0 dB = 1 inch^2/Hz

-10 dB = .1 inch^2/Hz

-20 dB = 01 inch^2/Hz

etc.

Displacement power was obtained from the acceleration power measured by double-integrating digitally, or specifically by dividing by f^4 because:

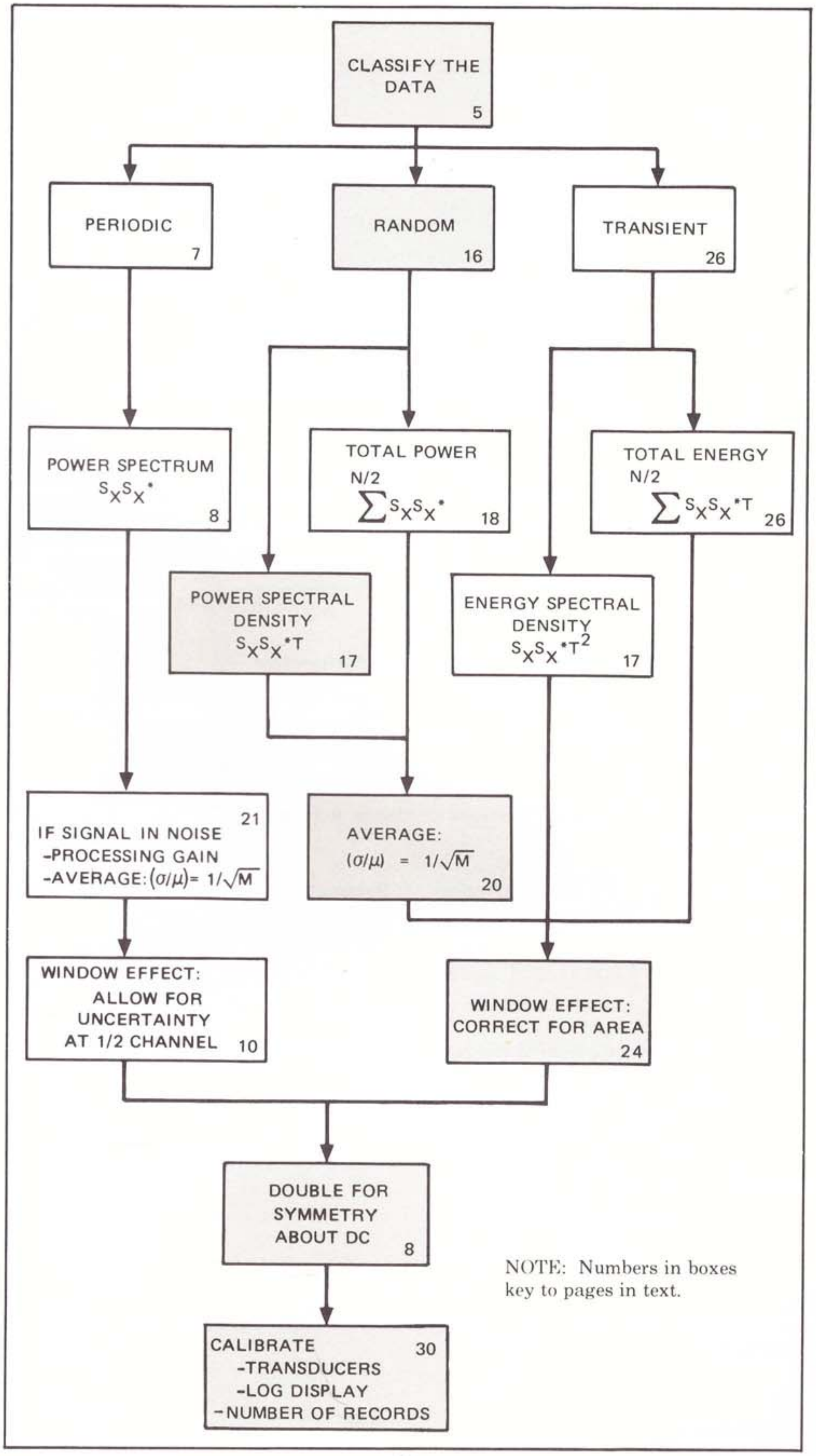
$$\text{displacement}^2 = \frac{\text{acceleration}^2}{\omega^4}$$

A correction factor in the keyboard program was used to provide the correct calibration.

[1]See reference 2 for a full discussion of aliasing.

[2]See reference 2

Table I. Measurement Method



The factor used in the keyboard program was achieved as follows:

(1) The displacement power spectrum was divided by the analysis bandwidth used 0.5 Hz to make a displacment power spectral density.

(2) The window used (two hannings) has an area under the square of the lineshape equal to 35/128.[1] This is also divided into the spectrum measured.

(3) To correct for symmetry about DC, the data is multiplied by 2.

(4) Because 400 records were ensemble summed, the data is divided by 400 to produce an ensemble average.

(5) To calibrate the displacement PSD in inches²/H, the data is multiplied by 9563 obtained from the following (X = displacement, A = acceleration):

$$X^2 = \left(\frac{A}{\omega^2}\right) = \frac{(A\ g/\text{volt})^2 \left(\frac{386.06\ \text{in/sec}^2}{g}\right)^2}{16\pi^4 \qquad f^4}$$

Because the acceleration power spectrum was divided by f^4, the above reduces to:

$$X^2 = 95.63\ A^2 \quad \frac{\text{inches}^2}{\text{volt}^2}$$

The accelerometer calibration A was 10 g/volt, therefore

$$X^2 = 9563 \quad \frac{\text{inches}^2}{\text{volt}^2}$$

Combining all of the above factors produces a factor of 349.7 used in the keyboard porgram:

Factor	Δf Bandwidth		Window		Symmetry		Number of Samples		Calibration
349.7 =	$\left(\frac{1}{.5}\right)$	•	$\left(\frac{128}{35}\right)$	•	(2)	•	$\left(\frac{1}{400}\right)$	•	(9563)

The results have a statistical certainty based on the 400 ensembles, or:

$$\frac{\sigma}{\mu} = \frac{1}{\sqrt{M}} = \frac{1}{\sqrt{400}} = \frac{1}{20}$$

For a 95% confidence level,

$$\frac{1.96\sigma}{\mu} = \frac{1.96}{20} = 9.8\%$$

Therefore, it can be said with 95% confidence that the results are ±9.8% (within 1/2 dB).

[1]See reference 1

If, for example (Figure 38), a broad peak centered at 7 Hz is interesting, integration of the PSD plot over specific limits will allow definition of the motion within that band.

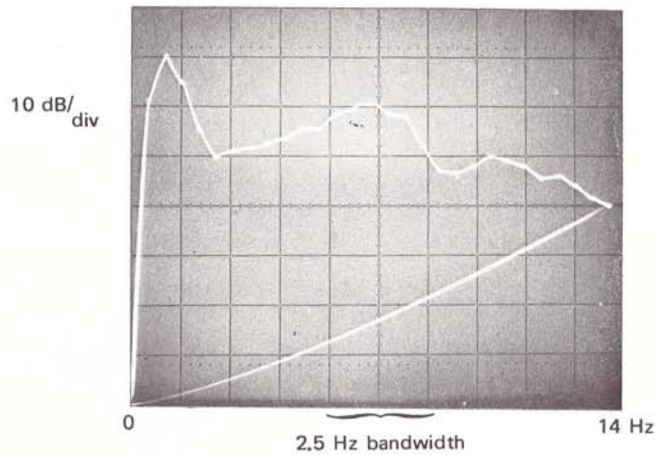


Figure 38

Summing the PSD over five spectral lines about the 7 Hz line results in .064 $inches^2$ total.

This corresponds to a motion of

$$.253\ inches_{rms}$$

for the 2.5 Hz bandwidth around 7 Hz.

REFERENCES

1. Potter, R.W., "Compilation of Time Windows and Line Shapes for Fourier Analysis", Hewlett-Packard, December 1971.

2. Carlson, John, "Fourier Analyzer Training Manual", AN 140-0, Hewlett-Packard, April 1970, Page 4-13.

3. Papoulis, Athanasios, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, Inc., 1965.

APPENDIX I

Window implementation.

Several of the windows in Reference 1 may be keyboard-called when User Program 3202 is made part of the Fourier system. A typical call is

USER PROGRAM 3202 SPACE 1 SPACE 301

where window P301 is desired in data block 1

This and/or other user programs may be merged with the Fourier system with the System Overlay Generator. See the HP Fourier Analyzer User Program Library for a list of available programs.

APPENDIX II
POWER SPECTRUM ENSEMBLE AVERAGE OF 100 SAMPLES

PROGRAM COMMANDS	CONTENTS BLOCK 0	CONTENTS BLOCK 1	PURPOSE OF COMMAND
LABEL 0			Establish Program Start
CLEAR 1		Cleared	Clear Sum Block
LABEL 1		Cleared	Establish Loop Label
ANALOG IN 0 SPACE 1	Current Time Record	Cleared	Acquire Data, Display Sum
FOURIER	Freq Spectrum of Data	Cleared	Transform Data
POWER SPECTRUM	Freq Spectrum of Data	Power Spectrum Sum of Data Samples	Compute Power Spectrum and Ensemble Sum 100 Samples
COUNT 1 SPACE 100	Freq Spectrum of Data	Power Spectrum Sum of Data Samples	
MULTIPLY 1 SPACE 2		Power Spectrum Sum of Data Samples	Correct for Symmetry
DIVIDE 1 SPACE 100		Power Spectrum Ensemble Average	Compute Average
LOG MAG 1		Power Spectrum Ensemble Average	Log Magnitude Data
END		Power Spectrum Ensemble Average	END

Block Size = 512, Δf = 1 Hz }
Block Size = 256, Δf = 2 Hz } F_{max} = 256 Hz
Display Horizontal Sweep 10.24 cm

APPENDIX III

POWER SPECTRAL DENSITY ENSEMBLE AVERAGE OF 400 SAMPLES

PROGRAM COMMANDS	CONTENTS BLOCK 0	CONTENTS BLOCK 1	PURPOSE OF COMMAND
LABEL 0			Establish Program Start
CLEAR 1		Cleared	Clear Summing Block
LABEL 1			Establish Loop Label
ANALOG IN 0 SPACE 1	Current Time Record		Acquire Data, Display Sum
FOURIER	Freq Spectrum of Data		Transform Data
POWER SPECTRUM		Power Spectrum Sum of Data	Compute Power Spectrum and Ensemble Sum 400 Samples
COUNT 1 SPACE 400		Power Spectrum Sum of Data	
MULTIPLY 1 SPACE 2		Power Spectrum Sum of Data	Correct for Symmetry
DIVIDE 1 SPACE 400		Power Spectrum Ensemble Avg.	Compute Average
DIVIDE 1 SPACE 10		PSD Ensemble Average	Compute Power Spectral Density
LOG MAG 1		PSD Ensemble Average	Log Magnitude of Data
END		PSD Ensemble Average	END

Block Size = 256 } F_{max} = 1280 Hz
Δf = 10 Hz }

Display Horizontal Sweep 12.8 cm

APPENDIX IV

An alternate method of calculating the total power (or energy) of a signal besides the power spectrum is the use of channel zero of the autocorrelation function. A detailed discussion of this may be found in Reference 2.

APPENDIX V
EXAMPLE MEASUREMENT PROGRAM

PROGRAM COMMANDS	CONTENTS BLOCK 0	CONTENTS BLOCK 1	CONTENTS BLOCK 2	PURPOSE OF COMMAND
LABEL 0			f^4	Establish Program Start
CLEAR 1		Cleared		Clear Summing Block
LABEL 1				Establish Loop Label
ANALOG IN	Current Time Record			Acquire Data
HANN	"			Window Data
HANN	"			
FOURIER	Frequency Spectrum of Data			Transform Data
POWER SPECTRUM		Power Spectrum Sum of Data		Compute Power Spectrum and Ensemble Sum 400 Samples
COUNT 1 SPACE 400		"		
DIVIDE 2	*Displacement Power Spectral Ensemble Sum*			Convert Acceleration Spectrum to a Displacement Spectrum
MULTIPLY 0 SPACE 3497	*Calibrated Displacement PSD* Ensemble Average			Calibrate Results
DIVIDE 0 SPACE 10				
LOG MAG	"			Log Magnitude of Data
DISPLAY 0 SPACE 0 SPACE 200	"			Expanded Display to show DC to 100 Hz
END	"			End

Blocksize = 1024 F_{max} = 256 Hz
Δf = 0.5 Hz

The function f^4 may be calculated easily and stored in a data block (block 2) for ready division into the acceleration power spectrum. The following keyboard steps will produce f^4 (note that the value entered is equal to the resolution Δf):

KEYBOARD COMMAND	CONTENTS BLOCK 0	CONTENTS BLOCK 2	PURPOSE OF COMMAND
CLEAR	Cleared		
KEYBOARD 0 SPACE 1 SPACE 512	"		Block fill Channel 2 through 512
KEYBOARD - 1 SPACE 4	"		10^{-1} Scale factor in Rect. Freq. Domain
5	0.5 in each channel		5×10^{-1}
INTEGRATE	f		Calculate f for each channel
MULTIPLY	f^2		Calculate f^2 for each channel
MULTIPLY	f^4		Calculate f^4 for each channel
STORE 2	f^4	f^4	Store for Future Divide Operation

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