# Errata

**Document Title:** Improving the Accuracy of Structural Response Measurements (AN 240-2)

**Part Number:** 5952-7120

**Revision Date:** August 1977

## HP References in this Application Note

This application note may contain references to HP or Hewlett-Packard. Please note that Hewlett-Packard's former test and measurement, semiconductor products and chemical analysis businesses are now part of Agilent Technologies. We have made no changes to this application note copy. The HP XXXX referred to in this document is now the Agilent XXXX. For example, model number HP8648A is now model number Agilent 8648A.

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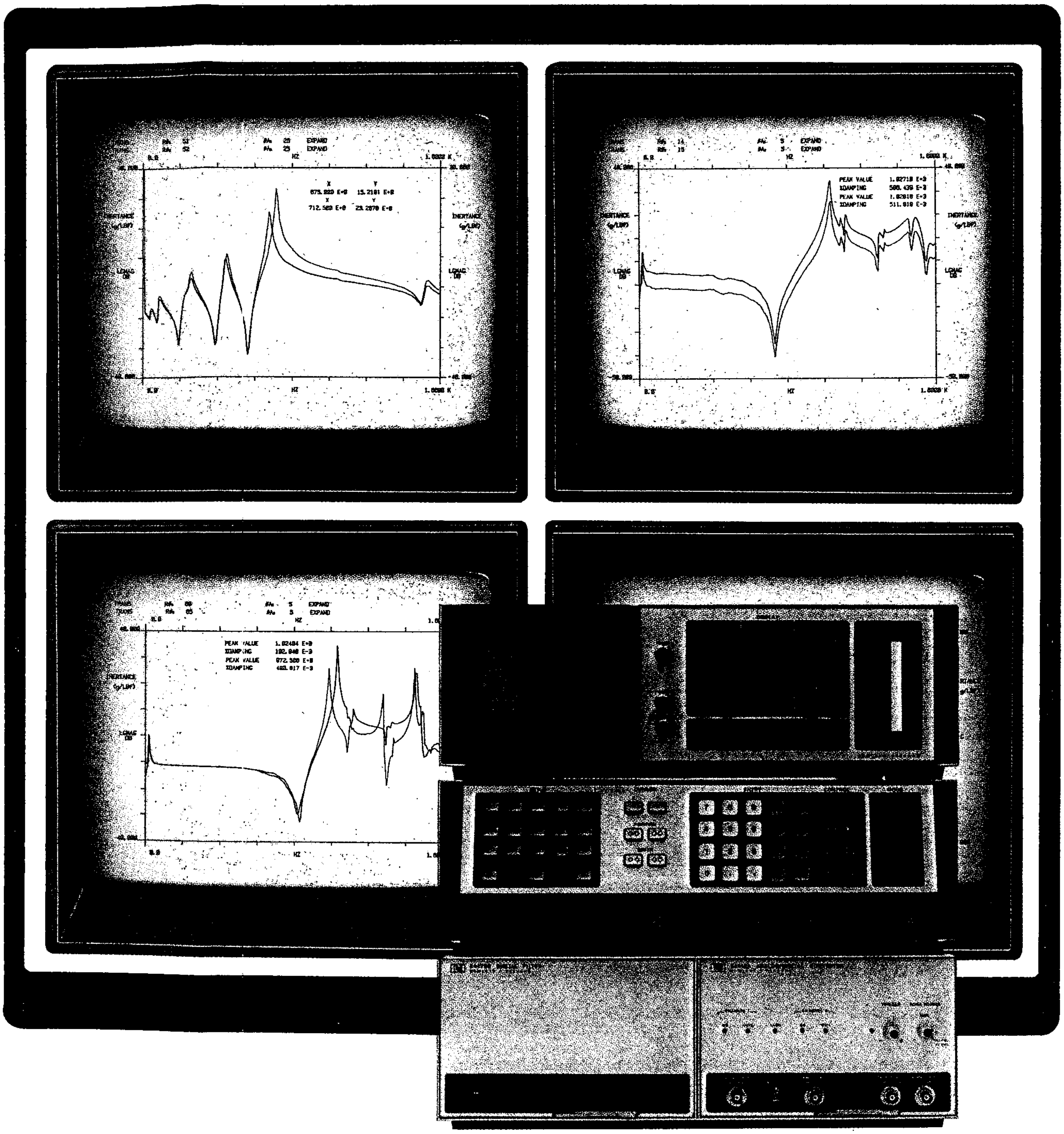
# APPLICATION NOTE 240-2

## Improving the Accuracy of Structural Response Measurements



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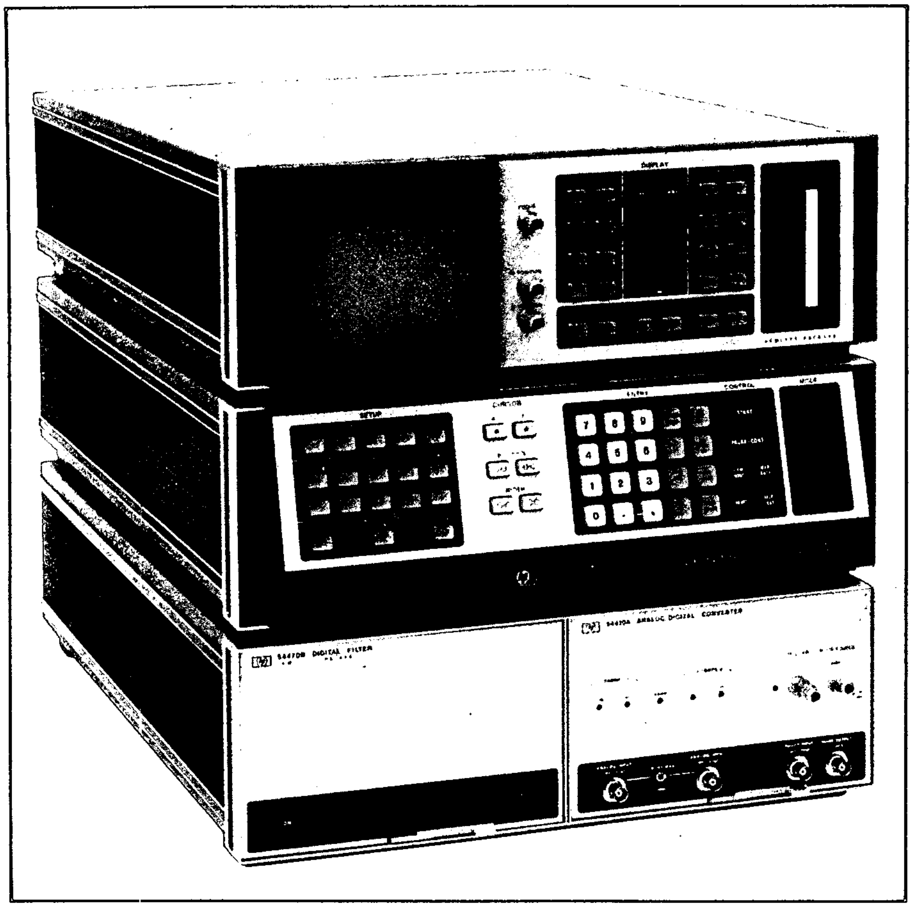

The Hewlett-Packard 5420A Digital Signal Analyzer utilizes sampled data techniques to perform a wide variety of single and dual channel measurements over a 25 KHz range. Measurements include Time Record Averaging, Auto and Cross Spectrum, Impulse Response, and Frequency Response. These measurements, and more, are described in Application Note 240-0.

An important feature of the HP 5420A is the Waveform Calculator which greatly enhances measurement and analysis capability by providing the user with the ability to mathematically operate on measured data. This feature allows easy computation of Coherent Output Power, non-Coherent Output Power, Equalized Transmissibility, Compliance, calibration in percent, and many more. The Mass Loading compensations described in this Application Note are also easily performed using the Waveform Calculator.

Other important features include Band Selectable Analysis, which provides excellent frequency resolution, and built in data storage on digital magnetic tape cartridges. The instrument also contains a random noise generator, which is useful as a test stimulus, and it provides a fully-annotated dual-trace display with flexible cursor capabilities. The 5420A may optionally be equipped to provide alphanumeric and graphic hardcopy directly on an HP 9872A Four Color Graphics Plotter or an HP 7245A Plotter/Printer.

# Identification and Correction of Mass Loading and Accelerometer Loading Errors

## INTRODUCTION

The measurement of frequency response is the heart of structural analysis. The frequency response, $H(\omega)$, is defined in terms of the single input/single output system, shown in Figure 1, as the ratio of the Fourier transform of the system output, $y(t)$, to the Fourier transform of the system input, $x(t)$.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

where $Y(\omega)$ = Fourier transform of system output, $y(t)$
$X(\omega)$ = Fourier transform of system input, $x(t)$

The frequency response function is complex — that is, it has associated with it both magnitude and phase (or, equivalently, a real and imaginary part). An example of the log magnitude display of a structural frequency response is shown in Figure 6, in which:

$$H(\omega) = \frac{A(\omega)}{F(\omega)}$$

where, $F(\omega)$ = Fourier transform of force input, $f(t)$
$A(\omega)$ = Fourier transform of acceleration response, $a(t)$

In this form structural resonances occur as peaks which represent frequencies of maximum dynamic weakness.

Two important, yet subtle, problems which affect frequency response measurements are MASS LOADING and ACCELEROMETER LOADING. Both of these problems are related in that their effects can lead to significant resonant frequency, damping, and amplitude errors. In extreme cases, these errors can lead to an incorrect solution for the problem being studied. Fortunately it is easy to identify and compensate or correct for these problems using a digital signal analyzer, such as the HP 5420A, with post-processing capability to mathematically operate on measured data.

## MASS LOADING

Loading problems arise because it is impossible to create the "ideal" measurement system in which the test structure appears on only one side of the load cell used to measure the input force, with the excitor (shaker or hammer) only on the other side of the load cell (see Figure 2).

In most practical cases, some mass will appear *between* the load cell and the test system. This causes the force measured by the load cell to differ from the force actually applied to the system. Figure 3 describes how loading can alter the input force since, to the load cell, the loading mass appears as part of the test system. Equation 1 implies that the force input to the system will be less than the force measured.

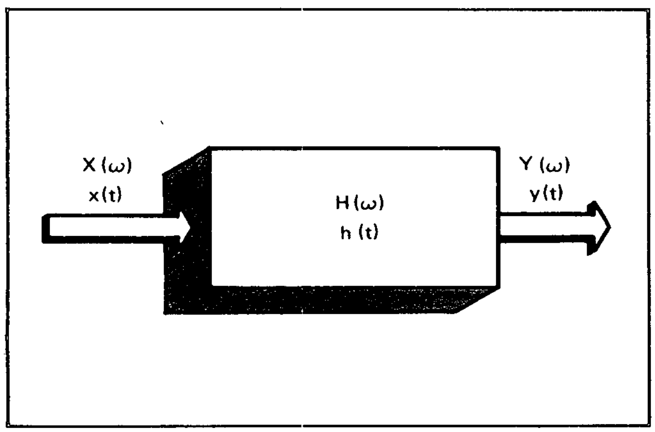


**FIG 1.** SINGLE INPUT/SINGLE OUTPUT SYSTEM

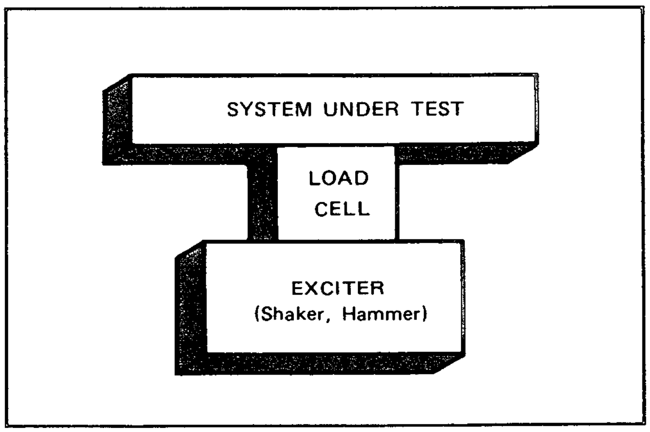


**FIG 2. "IDEAL" MEASUREMENT SET-UP — Test system and exciter appear on separate sides of the load cell. Force sensed by load cell is the force actually applied to the structure.**

The increase in effective system mass also causes a downward shift of resonant frequencies. The amount of shift for a single degree of freedom system is given by:

(EQ.2)

$$\Delta f_N = f_N \left(1 - \sqrt{\frac{M_S}{M_A + M_S}}\right)$$

where, $f_N$ = resonant frequency with no loading
$\Delta f_N$ = change in resonance frequency
$M_S$ = static mass of structure
$M_A$ = added mass due to loading

Equations 1 and 2 suggest that loading errors will be minimized if the additional mass is much smaller than that of the test structure. While this is true, it is important to realize that at certain frequencies (resonances) the effective mass of that mode will often be much lower than the system rest mass. Therefore, loading errors may be significant, especially for lightly damped resonances, even though the loading mass is quite small.

Figure 4 shows how mass loading often occurs in both shaker and impact test set-ups. The mass used to mount the structure to the load cell causes loading when using a shaker to supply the input force (Figure 4A). In the case of impact testing (Figure 4B), loading results from the mass of the hammer tip used to deliver the input force to the structure.

The correction procedures differ depending upon the type of excitation used (shaker or impact) and will be discussed separately.

## CORRECTING FOR MASS LOADING

**Shaker Test System** The results derived in Figure 3 can be used to derive a general compensation formula for removing the mass loading errors that arise in a shaker test system.

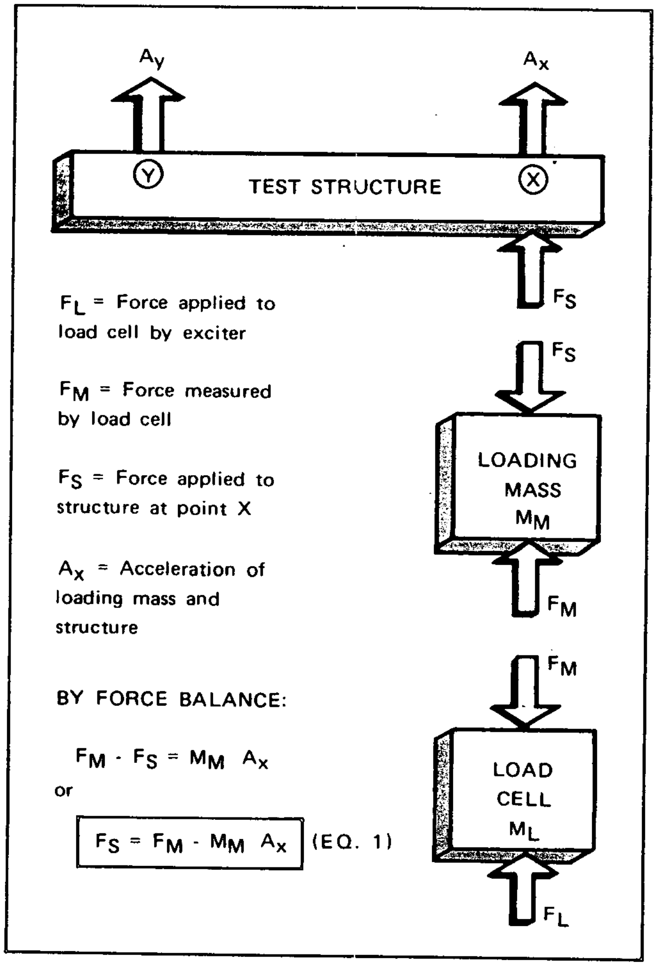


**FIG 3. FORCES ACTING ON THE TEST SYSTEM — Equation 1 shows that the force actually applied to the structure is less than the force measured due to loading.**

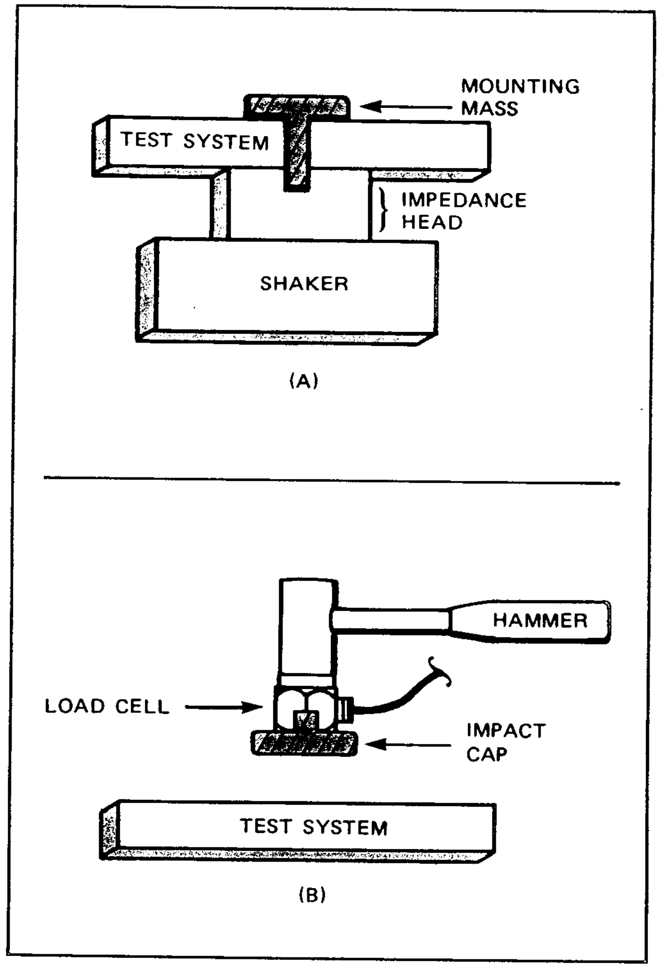


**FIG 4. "REAL" MEASUREMENT SET-UPS — (A) Shaker test system, loading caused by mounting screw. (B) Impact test system, loading caused by impact cap and hardware. Force sensed by load cell differs from force applied to structure due to additional mass.**

The first step is to measure the frequency response of the mounting device using an impedance head as shown in Figure 5. The resulting measurement is

*(EQ. 3)*

$$H_I = \frac{A_I}{F_I} = \frac{1}{M_M}$$

where $M_M$ = effective loading mass
$A_I$ = acceleration response of impedance head
$F_I$ = input force

Using the same mounting device, the test structure is then attached to the impedance head (see Figure 4A). The resulting frequency response is the uncompensated driving point measurement (refer to Figure 3),

*(EQ. 4)*

$$H_{xx}' = \frac{A_x}{F_M}$$

In general, the measured frequency response between points X and Y on the structure is

$$H_{xy}' = \frac{A_y}{F_M}$$

where $F_M$ is the input force measured by the load cell. However, the *actual* frequency response between those points is

$$H_{xy} = \frac{A_y}{F_S}$$

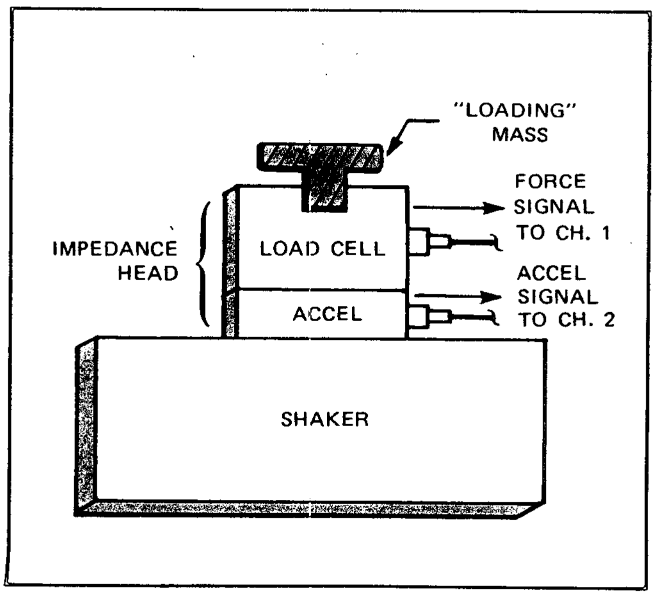


**FIG 5. Test set-up used to measure effective mass of impedance head with loading mass. Resulting frequency response is $H_I$.**

where $F_S$ is the actual input force.

Using equation 1,

$$F_S = F_M - M_M A_x$$

it is possible to determine $H_{xy}$ in terms of measureable functions as follows:

$$H_{xy} = \frac{A_y}{F_S} = \frac{A_y}{F_M - M_M A_x}$$

$$= \frac{A_y/F_M}{1 - (M_M A_x/F_M)}$$

$$= \frac{(A_y/F_M)\,(1/M_M)}{(1/M_M) - (A_x/F_M)}$$

or, substituting EQS. 3 and 4, the actual frequency response between points X and Y is:

*(EQ. 5)*

$$\boxed{H_{xy} = \frac{H_{xy}'\,H_I}{H_I - H_{xx}'}}$$

An example of a frequency response measurement before and after compensation is shown in Figure 6. In this case loading was caused by a small screw used to mount a beam to the impedance head (see Figure 4A). Note the large amplitude and frequency errors caused by mass loading for the lightly damped resonance near 700 Hz. The resonant frequency before compensation is 675 Hz, which is 37.5 Hz lower than the corrected value of 712.5 Hz. There is also an amplitude error of nearly 10 db.

A detailed procedure for performing this compensation using the HP 5420A Digital Signal Analyzer is given on page 8, APPENDIX A.

## CORRECTING FOR MASS LOADING

**Impact Test System** In an impact test mass loading occurs because the force measured by the load cell is not the force actually applied to the structure (as described in Figure 3). Figure 4B shows a typical test set-up with loading caused by the impact cap mounted onto the load cell and hammer.

One method of compensating for this additional mass is to construct a simple calibration system and calculate a "calibration" curve for the hammer, load cell, accelerometer, and impact cap used to measure the test structure. This curve can then be used to correct all subsequent measurements for loading errors.

This compensation method differs from that used in the shaker set-up because it is not possible to measure the frequency response of just the loading mass without using an external reference. The calibration system serves as the reference from which it will be possible to determine the dynamics of the loading mass.

The calibration system is nothing more than a relatively large known mass suspended on a long support (see Figure 8). If the suspension is of sufficient length and the motion of the mass when excited along its center of gravity is small, then the motion will approach a single degree of freedom.

Assuming the calibration system shown in Figure 8 is a single degree of freedom free mass system, we know from Newton's Law that

$$F = M_C A$$

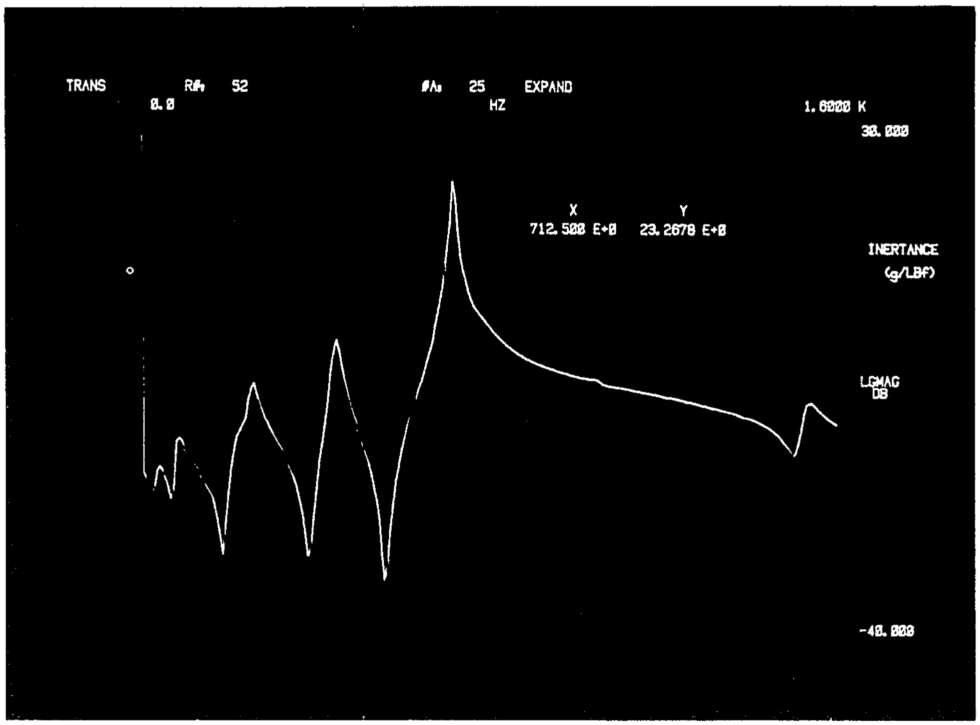


**FIG 6. MASS LOADING EFFECTS — Shaker Test System — White trace shows driving point transfer function after compensation, Black trace shows uncompensated measurement. Note that mode frequencies and amplitudes are lower due to mass loading, as indicated for large mode near 700 Hz.**

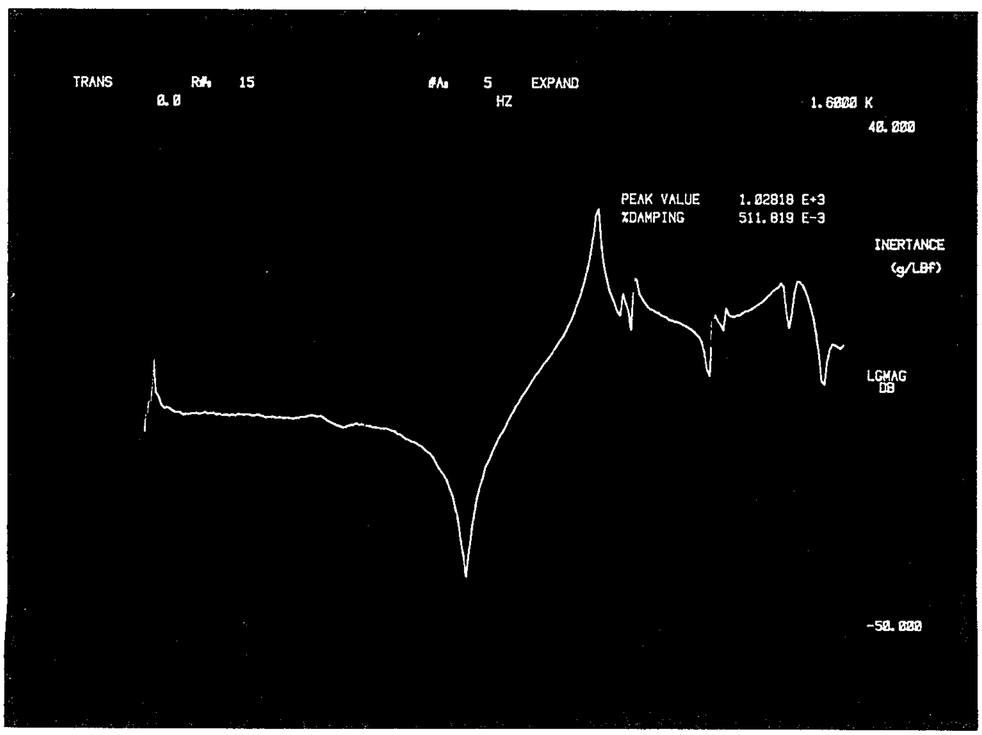


**FIG 7. MASS LOADING EFFECTS — Impact Test System — White trace shows representative transfer function after compensation, Black trace shows uncompensated measurement. Note that mode frequencies and dampings are slightly lower because of mass loading, as indicated for the mode near 1 KHz.**

or

*(EQ. 6)*

$$\frac{A}{F} = \frac{1}{M_C}$$

where $M_C$ = known calibration mass.

Equation 6 will hold for frequencies above the rigid body modes of the calibration mass and is the expected frequency response of the calibration mass. Any differences between the expected and measured frequency response are due to mass loading. Therefore, the calibration function of the hammer, load cell, accelerometer, and impact cap is simply the expected frequency response divided by the measured frequency response:

*(EQ. 7)*

$$H_C = \frac{1/M_C}{A_I/F_I} = \frac{1}{M_C H_I}$$

where,

$H_C$ = calibration function
$H_I$ = measured frequency response of calibration mass
$M_C$ = mass of free mass (in appropriate units)

If we now define the measured frequency response of the structure under test (see Figure 4B) to be

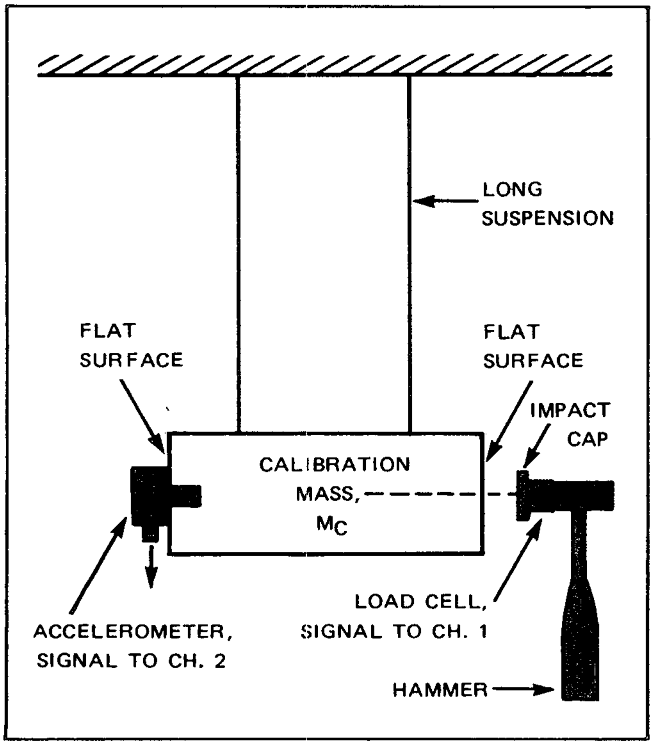


**FIG 8. CALIBRATION MASS SYSTEM — The mass, $M_C$, should be large with respect to the accelerometer used. To determine the calibration curve, the mass is impacted along its center of gravity.**

*(EQ. 8)*

$$H_M = \frac{A_M}{F_M}$$

where $A_M$ = measured acceleration
$F_M$ = measured force

then, assuming linearity and multiplying equations 7 and 8, the actual frequency response of the test structure is

*(EQ. 9)*

$$\boxed{H_A = H_C H_M}$$

Therefore, to correct impact test measurements, it is simply necessary to multiply the measured result by the calibration curve, $H_C$.

Calibration functions can also be calculated for use with velocity or displacement transducers. Equation 7 applies for acceleration transducers and can be modified by expressing acceleration in terms of velocity and displacement using the fact that differentiation in the time domain is equivalent to multiplication by $j\omega$ in the frequency domain. That is,

*(EQ. 10)*

$$\begin{array}{llllll} \text{TIME DOMAIN:} & a(t) & = \frac{d}{dt} & v(t) & = \frac{d^2}{dt^2} & x(t) \\ & \updownarrow & & \updownarrow & & \updownarrow \\ & \text{ACCEL} & & \text{VEL} & & \text{DISPL} \\ & \updownarrow & & \updownarrow & & \updownarrow \\ \text{FREQ. DOMAIN:} & A(\omega) & = j\omega & V(\omega) & = (j\omega)^2 & X(\omega) \end{array}$$

For *velocity* transducers:

*[from EQ. 10]*

$$H_{A,vel} = \frac{1}{j\omega} H_{A,accel}$$

*[from EQ.9]*

$$= \frac{1}{j\omega}(H_{C,accel})(H_{M,accel})$$

*[from EQS. 7 & 10]*

$$= \frac{1}{j\omega}\,\frac{1}{M_C\, j\omega\,(H_{I,vel})}\, j\omega\,(H_{M,vel})$$

$$= \frac{1}{j\omega\, M_C\, H_{I,vel}} H_{M,vel}$$

where $H_{A,accel}$, $H_{C,accel}$ = functions derived if an accelerometer had been used

$H_{I,vel}$, $H_{M,vel}$ = functions measured using a velocity transducer

Therefore, the calibration function to use in EQ. 9 is:

*(EQ. 11)*

$$H_{C,vel} = \frac{1}{j\omega \; M_c \; H_{I,vel}}$$

where $H_{I,vel}$ = measured frequency response of calibration mass using a velocity transducer

Similarly, for *displacement* transducers the necessary calibration function is:

*(EQ. 12)*

$$H_{C,dspl} = \frac{1}{(j\omega)^2 M_c \; H_{I,dspl}}$$

where $H_{I,dspl}$ = measured frequency response of calibration mass using a displacement transducer

The ability of the HP 5420A to multiply by $j\omega$ and invert data makes it possible to perform the calculations in Equations 11 and 12.

Figure 7 shows a frequency response measurement from a disk brake assembly before and after compensation. In this case, loading caused the overall level to be slightly high and the measured resonant frequencies to be slightly low. The corrected measurement shows a resonant frequency of 1028 Hz and damping of 0.512%. The amplitude error occurs because the force being measured during the impact is greater than the force imput to the structure. The magnitude and phase of the calibration function, $H_C$, used in Figure 7 is shown in Figure 9.

A detailed procedure for performing the compensation using the HP 5420A Digital Signal Analyzer is given on page 9, APPENDIX B. The steps necessary to perform the calibration with velocity or displacement transducers are included.

## ACCELEROMETER LOADING

Accelerometer loading errors occur when the transducer used to measure the response of the test structure adds too much mass. The effects are similar to those caused by mass loading in that resonant frequencies are reduced (see EQ. 2) and dampings increased.

To minimize accelerometer loading errors, it is necessary to use a transducer which will not alter the test structure dynamics. A simple "rule of thumb" can be applied to determine if accelerometer loading is significant. The procedure is as follows and is applicable for either shaker or impact testing:

1. Measure a typical frequency response on the test system using the desired accelerometer. Save this measurement.
2. Mount a second mass equivalent to the accelerometer mass (e.g. a similar transducer) at the same location to double the loading. Repeat step 1.
3. Compare the two measurements as shown in Figures 10 and 11.

If the two measurements differ significantly, then accelerometer loading is a problem and a smaller

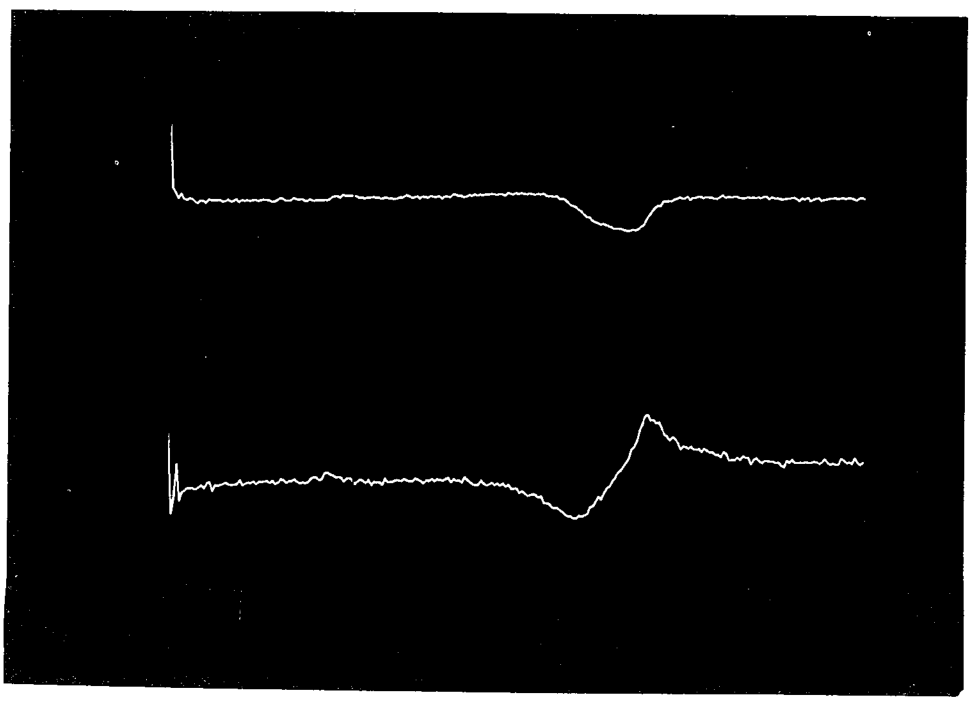

**FIG 9. $H_C$ FOR IMPACT TEST SYSTEM — Compensation curve calculated from calibration mass system.**

transducer should be used. On a very small structure it may even be necessary to measure the response using some type of acoustical or optical device which applies no load at all.

The measurements shown in Figures 10 and 11 were taken between the same two points on a disk brake assembly. Figure 10 shows that the initial accelerometer was too massive and altered the assembly's dynamics. Figure 11 shows that, using a smaller accelerometer, the loading errors were minimized.

Once a transducer is found which does not appreciably load the test structure, accurate system measurements can be performed.


**FIG 10.** EFFECTS OF ACCELEROMETER LOADING — **Accelerometer too massive — Black trace shows transfer function measurement with one accelerometer, White trace shows same measurement with accelerometer mass doubled. Comparison shows that this accelerometer was too massive for the disk brake structure under test.**

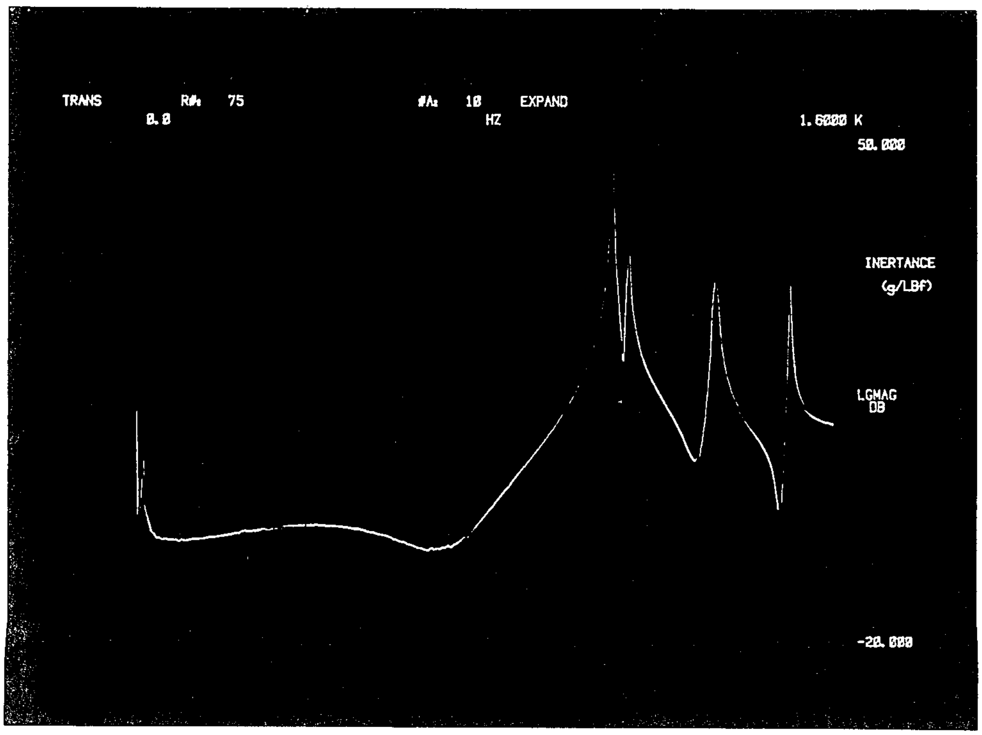


**FIG 11.** EFFECTS OF ACCELEROMETER LOADING — Accelerometer OK — **Same measurements as in Figure 10, but using a smaller accelerometer. In this case, accelerometer loading is insignificant.**

## SUMMARY

Frequency response studies of mechanical structures are often hampered by undesirable loading due to mass added by excitation hardware or response transducers. Usually, the existence of excessive loading is not obvious since the data obtained from the measurement looks perfectly good. The problem is that the data represents not only the structure of interest, but also the effects due to the additional mass. These loading effects can be quite pronounced and, if not detected, can obscure the solution to the problem being studied.

Modern digital signal analyzers such as the HP 5420A, which provide *post-measurement processing* can be used to test for the presence of loading. In cases where loading is significant, suitable corrections or modifications can be applied. Once corrected, the data can be utilized to aid in the problem solution since it will accurately describe the system dynamics.

# Mass Loading Compensation Using the HP 5420A Digital Signal Analyzer

## APPENDIX A

### Shaker Test System

1. Measure $H_I$ using the set-up shown in Figure 5.
2. SAVE $H_I$ on the data cartridge in a record number higher than six (e.g. record #10).
3. Attach the structure to the impedance head using the same mounting device and measure $H_{11}'$.
4. SAVE $H_{11}'$ in a record number higher than six (e.g. record # 11).
5. Enter 0 RESET to clear the memory data space.
6. RECALL $H_I$ onto both TRACE A and TRACE B.
7. With TRACE A active enter the $H_{11}'$ data record number followed by × (multiply). This will create the product $H_{11}' H_I$ on TRACE A.
8. With TRACE B active, enter the $H_{11}'$ data record number followed by − (subtract). This creates the difference $H_I - H_{11}'$ on TRACE B.
9. Now make TRACE A active and enter ÷ (divide). TRACE A now contains the *actual* system driving point response, $H_{11}$.
10. RECALL $H_{11}'$ onto TRACE B and compare to $H_{11}$ using FRONT/BACK FORMAT and the same vertical scale (see Figure 6).

If the loading errors are significant, further system measurements can be corrected as follows:

11. Measure the frequency response between points 1 and 2 on the structure, $H_{12}'$.
12. SAVE $H_{12}'$ in a record number higher than six.
13. Enter 0 RESET.
14. RECALL $H_I$ onto both TRACE A and TRACE B.
15. With TRACE A active, enter the $H_{12}'$ data record number followed by × (multiply). This will create the product $H_{12}' H_I$ on TRACE A.
16. With TRACE B active, enter the $H_{11}'$ data record number followed by − (subtract). This creates the difference $H_I - H_{11}'$ on TRACE B.
17. With TRACE A active, enter ÷ (divide). TRACE A now contains the corrected frequency response, $H_{12}$.

## APPENDIX B

### Impact Test System

1. Measure the frequency response of the calibration mass, $H_I$, using the hammer, load cell accelerometer, and impact cap that will be used to measure the test structure. The accelerometer is mounted at one end of the mass and impacted at the other end as shown in Figure 8.
2. SAVE $H_I$ on the 5420A data cartridge.
3. Enter 0 RESET
4. RECALL $H_I$ onto both TRACE A and TRACE B.
5. Create the constant $M_C$, the known calibration mass in the appropriate units (e.g. lbm or kgm), on TRACE B using the command, $M_C$, 0 RECALL.
6. With TRACE B active, enter × (multiply). This will create the product $M_C H_I$ on TRACE B.
7. If $H_I$ was measured using *velocity* or *displacement* transducers, operate on TRACE B using the $j\omega$ key once (EQ. 11) or twice (EQ. 12).
8. With TRACE B still active, enter — 1 ÷ (divide). This creates the calibration function, $H_C$, on TRACE B.
9. SAVE $H_C$ on the data cartridge in a record number higher than six.
10. Measure a frequency response on the test structure using the same hammer, load cell, accelerometer, and impact cap used to measure $H_I$. This is $H_M$ (see EQ. 8).
11. With $H_M$ on both TRACE A and TRACE B and TRACE B active, enter the $H_C$ record number followed by × (multiply). This will create the *actual* structure frequency response, $H_A$, on TRACE B. $H_M$ remains on TRACE A.
12. To compare $H_M$ and $H_A$, use FRONT/BACK FORMAT and expand both traces over the same range (see Figure 7).

If loading errors are deemed significant, all further measurements can be corrected using step 11 to multiply by $H_C$.


**References**

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