HEWLETT
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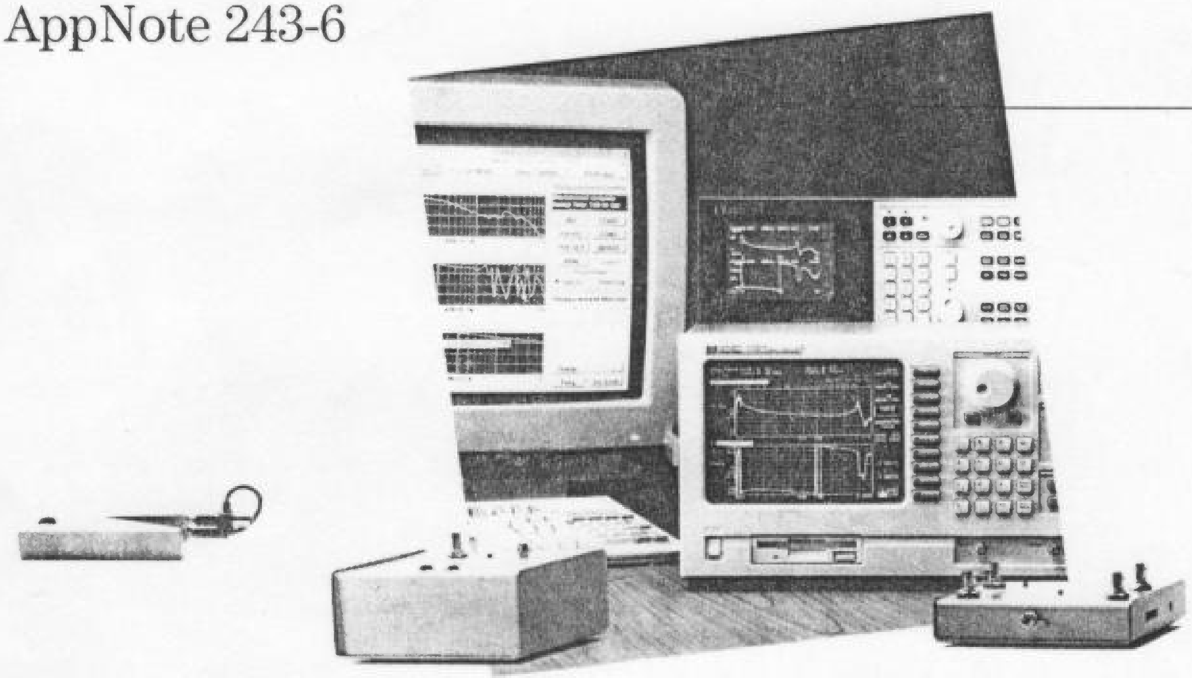
Control System Measurement Fundamentals Using Dynamic Signal Analyzers and Accessories

AppNote 243-6



Overview

Control systems are becoming more and more common and can be found in many of today's consumer products, as well as very technically sophisticated products. Closed-loop control systems are used in compact disc (CD) players, electronic fuel injection systems in automobiles, and within the pen positioning systems of low cost plotters. Complex closed-loop control systems have traditionally been used in aerospace applications such as the aerodynamic control surface systems in combat fighters or missiles.

Historically, testing control systems during the development cycle has been difficult because the desired result is the open-loop frequency response characteristic and the loop must remain closed to achieve stable operation. The open-loop characteristic, sometimes called the loop-gain characteristic, is an important measurement since it is used in defining, as well as refining, a mathematical model for the control system. It is also used to design stability compensation networks and determine stability parameters such as gain and phase margins.

The use of modern dynamic signal analyzers for studying the dynamic performance characteristics of control systems has gained considerable acceptance over the last several years. This application note reviews the basic measurement methods and discusses in detail some aspects of good, accurate measurements in a variety of situations.

Unless otherwise noted, all lower case letters in the following paragraphs denote variables or functions in the time domain, and upper case variables represent transformed quantities. These quantities may be either Laplace transformed (i.e., $G(s)$) as a function of the complex variable s $(s = \sigma + j\omega)$ or Fourier transformed (i.e., $G(j\omega)$) as a function of the frequency variable $j\omega$

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Section 1

Control System Review

A simple feedback control system model is shown in figure 1.

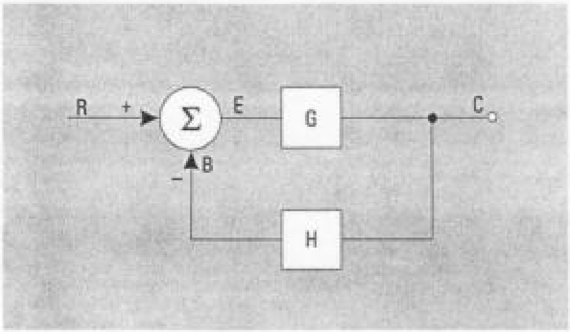


Figure 1: Simple feedback control system

From this simple model we can derive the basic equation of the closed-loop control system:

$C = EG$ and $E = R - B$

$C = (R - B)G$

$C = RG - BG$, but $B = CH$

$C = RG - CHG$

$C + CHG = RG$

$C(1 + GH) = RG$

$$\frac{C}{R} = \frac{G}{1 + GH}$$

GH is the open-loop gain function. A simple evaluation of this equation shows that:

If $GH \to -1$, then: $\frac{C}{R} \to \infty$

As was shown by H. Nyquist in the 1930's, GH provides a fundamental and straightforward way to describe and understand the stability of a control system. For an analog control system, the block diagram in figure 1 is shown with the details of a Laplace transform point of view, as in figure 2. For a digital control system the block diagram typically is expressed in a Z-transform.

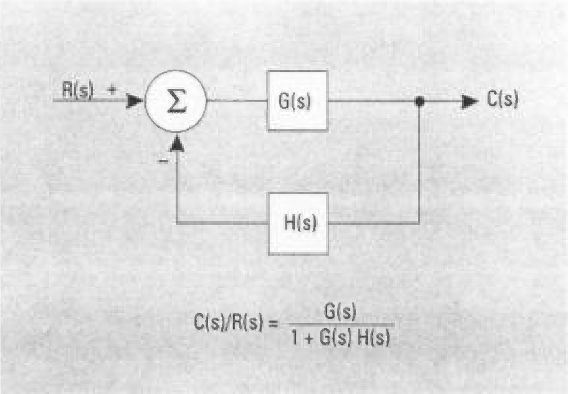


Figure 2: Closed-loop transfer function

From a control system perspective the equations for the system open-loop gain are typically expressed using Laplace transform notation. This simplifies the analysis by, turning the differential equations into simple arithmetic operations to obtain solutions. These Laplace domain equations are either in polynomial or pole-zero form as shown in figure 3.

$$\frac{C(s)}{R(s)} = \frac{s + 3.5}{s^3 + 14s^2 + 89s + 780}$$
$$= \frac{s + 3.5}{(s + 12)(s + 1 + j8)(s + 1 - j8)}$$

Figure 3: System equations

Typically, the pole-residue form is not used in control system work. The pole-zero form is obtained by solving for the roots of both the numerator and denominator of the polynomial form. Some of today's frequency response analyzers can work directly with these analytical functions. This is useful, for predicting the closed-loop performance with compensation networks using measured open-loop gains and analytical frequency responses of the compensation networks.

Figure 4 shows the poles and zeros of the equations in figure 3 plotted in the complex Laplace (s) plane. The roots of the numerator in figure 3 give a single zero at s = -3.5, while the roots of the denominator give a pole at s = -12 and a complex conjugate pair of roots at s = -1 + j8 and s = -1 - j8.

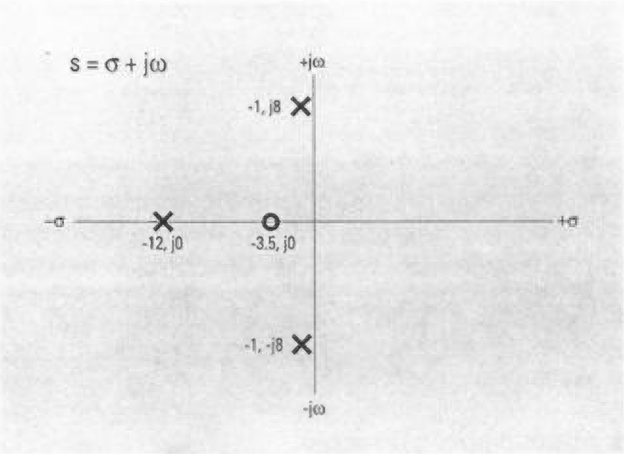


Figure 4: Complex Laplace(s) plane

The s-domain plots of this stable, closed-loop system are shown in figures 5 and 6.

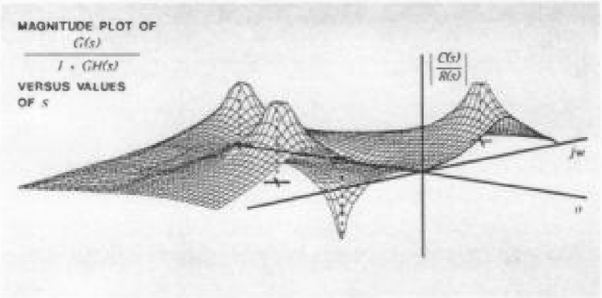


Figure 5: Transfer function magnitude

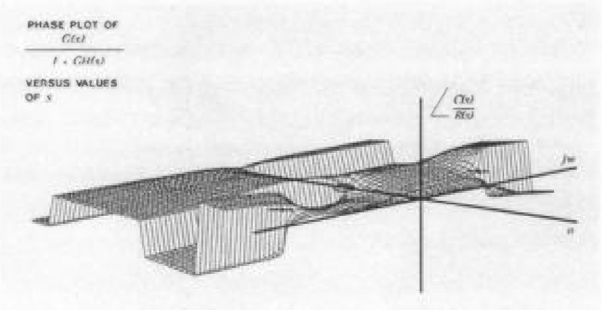


Figure 6: Transfer function phase

From Nyquist's study of the closed-loop transfer functions, he observed that for a stable system there could be no poles in the right half s-plane. This relationship constitutes Nyquist's stability criteria. Calculating the poles of the closed-loop transfer function involves some work (especially in the 1930's), so working with the open-loop function directly saved some labor. From a stability point of view, Nyquist presented the concept of graphing the open-loop gain function and observing whether it enclosed the point -1, j0 in the complex plane. In figure 7, there are two different open-loop gain functions plotted; one system plot encloses this point and would therefore become an unstable system if the loop was closed.

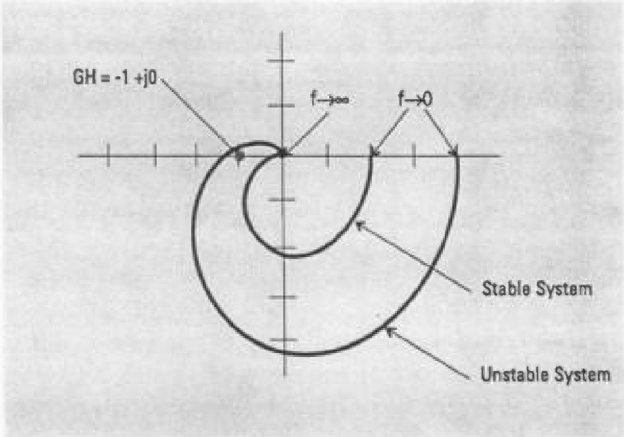


Figure 7: Stability and nyquist Plot

These criteria enable one to determine whether a system is stable or unstable, but the relative degree of stability is a more desired measurement. H. Bode in 1940 presented this concept in the form of gain and phase margins. Figures 8 and 9 present this relative stability criteria in a graphical perspective. The gain margin is simply the reciprocal of the gain where the open-loop frequency response function's phase is at minus 180 degrees. The phase margin is equal to 180 degrees minus the phase of the open-loop frequency response at the point where the gain is unity (or 0 dB). Traditionally, a system that has less than 30 degrees of phase margin or less than 6 dB (a gain factor of 2.0) of gain margin is considered marginal or possibly unacceptable.

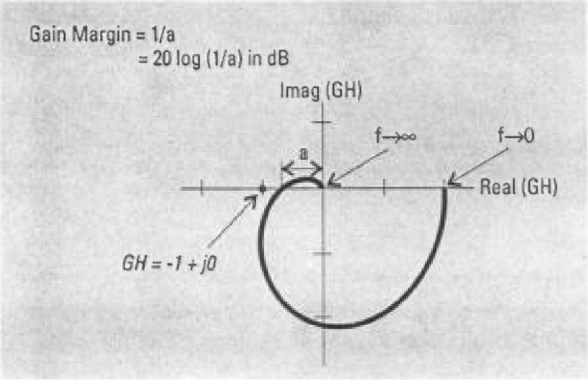


Figure 8: Relative stability gain margin

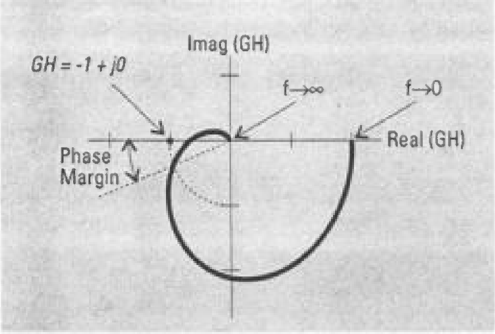


Figure 9: Relative stability phase margin

Instead of working from a polar coordinate perspective, the same gain and phase margins can be obtained from a plot of log magnitude-versus-phase, called a Nichols Diagram. The disadvantage of both the Nyquist and Nichols plots is that frequency information is more difficult to ascertain in these single plot formats.

Another, way of looking at the gain and phase margins that preserves the frequency information is the Bode plot (a plot of log magnitude-versus-frequency and phase-versus-frequency), shown in figure 10.

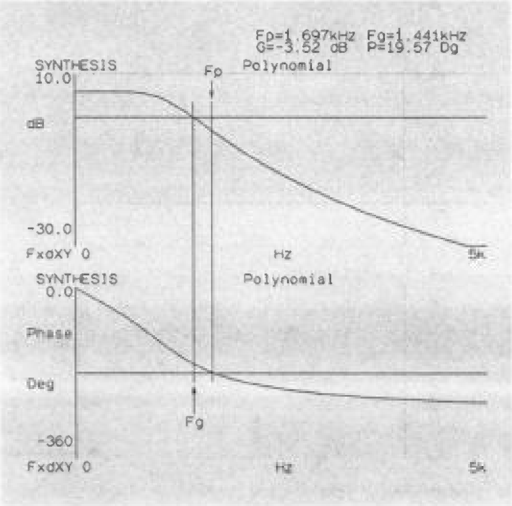


Figure 10: Bode plot

Section 2

Measurement Background

The basic measurement process begins with determining the open-loop characteristic. Using a network or dynamic signal analyzer a logical approach is to break the loop as shown in figure 11, and directly measure the open-loop gain Y/Z. While conceptually correct, this approach is doomed with most control systems because a simple position servo with no feedback will ramp to one or the other stops and saturate the amplifiers in a futile effort to control the system. However, this approach might be quite satisfactory on simple systems that are not controlling large physical devices.

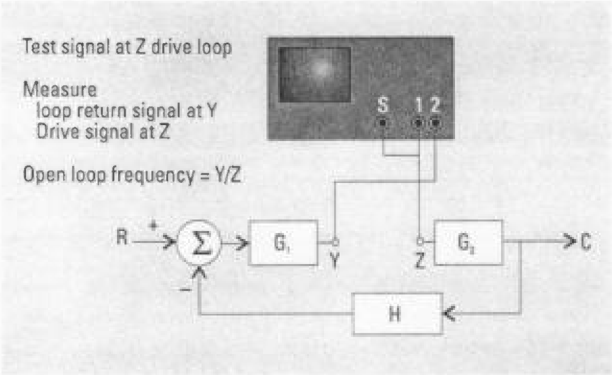


Figure 11: Open-loop direct Y/Z

Instead of attempting to measure the open-loop characteristic directly by opening the loop, this application note will address measuring the system with the loop closed and calculating the open-loop characteristic from a measurement with the loop closed.

Many combinations of measurements can be made on a closed-loop system. Figure 12 shows one system model which can give rise to three different signal pairs from which the desired estimate of the open-loop gain function, GH, can be determined (Z/S, Y/Z & Y/S).

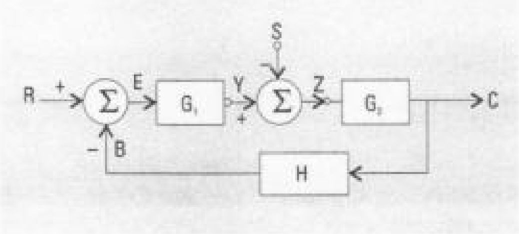


Figure 12: Measurement setup—general internal case

This model is referred to as "internal" because the added summing junction has been placed inside the control loop. We can also define two different measurement methods, one based upon the fast Fourier transform (FFT) and a second method based upon the swept Fourier transform (SFT). Each of these methods require an excitation signal that is consistent with the method. For the FFT method, a broad band excitation is needed (such as random noise). For the SFT method, a slow swept-sine excitation is required. This "internal" excitation method therefore provides six different ways of estimating the desired loop gain function GH. An additional internal method is described in HP Application Note 243-5, "Control System Loop Gain Measurements". That note discusses 18 ways to make these measurements, ranks them as to which should make better measurements, presents the calibration issues and procedures to improve the measurements, describes the expressions for calculating the true coherence function, and presents a table of the maximum loop gain magnitude that can be handled by each method.

Figure 13 represents the "external" summing junction which presents five more combinations of variable pairs to use (B/E, E/A, E/S, B/A & B/S).

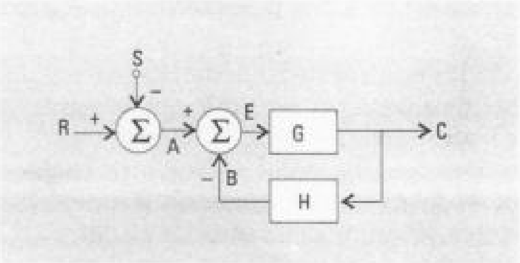


Figure 13: Measurement setup - general external case

Using the FFT and SFT measurement methods we arrive at ten additional ways of estimating the loop-gain function, GH. There is also another method called B+E which, theoretically, is the best method of all eighteen. The intent of this note is to introduce the new user to some the practical measurement methods and considerations in applying these concepts.

Signal Injection Devices

The measurement techniques described in this application note are based upon the addition of an auxiliary, unity-gain summing junction to the system. The ideal summing junction allows access to needed measurement points and at the same time allows the measurements to be made while the system remains in its normal closed-loop state, as shown in figure 14.

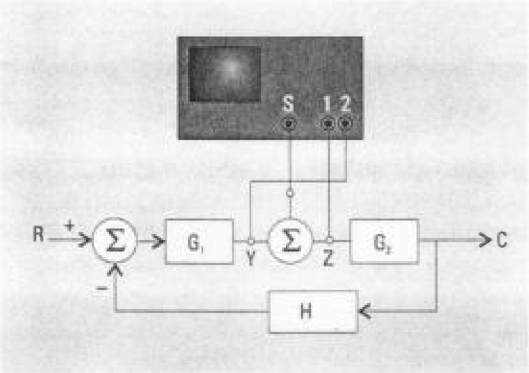
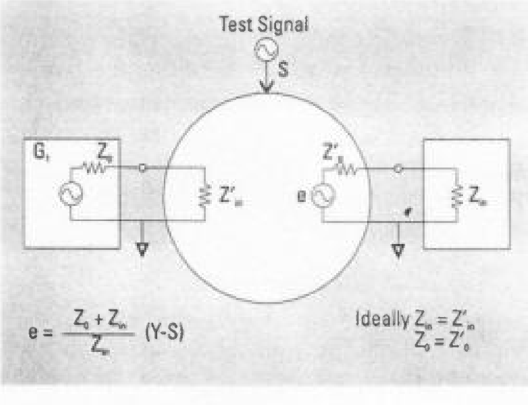


Figure 14: Measurement setup closed-loop direct (Y/Z)

This summing junction allows an external signal to be injected into the loop while providing the isolation required to ensure that the Y and Z signals are measurable with the control-loop closed. When installing this signal injection device into the circuit, one must consider how well matched the input and output impedances are at the location that the signal injection device was inserted. Figure 15 conceptually models this consideration. Ideally the impedances should be matched as closely as possible so that neither the gains nor the frequency response characteristics of the loop (G1 or G2) have been changed. In other words, inserting this network into the loop should not upset or change the control-loop response.

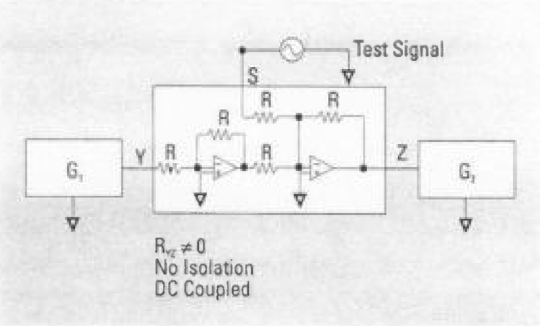


$e = \frac{Z_0 + Z_{in}}{Z_{in}}$ (Y-S)

Ideally $Z_{in} = Z'_{in}$
$Z_0 = Z'_0$

Figure 15: Impedance

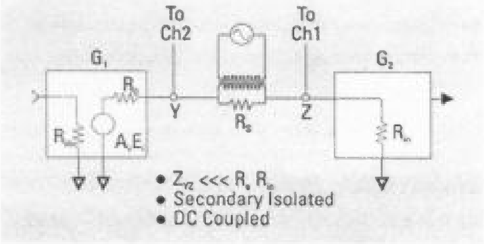
There are several devices that can be used as summing junctions. Figure 16 depicts a traditional operational amplifier summing junction whereby the source signal is buffered by the second amplifier. The overall input to output has a net gain of unity (+) since each operational amplifier inverts its signal by minus one. The impedance effects will be minimal if G1 in the control-loop circuit has an output impedance that is much less than the input impedance R of the summing junction (1:10,000) and the input impedance of G2 is similar to R.



$R_{YZ} \neq 0$
No Isolation
DC Coupled

Figure 16: Active injection device

Signal injection devices range from active networks to passive devices such as the transformers shown in figure 17.



- $Z_{YZ} <<< R_s R_{in}$
- Secondary Isolated
- DC Coupled

Figure 17: Transform signal injection

Each offers a set of characteristics for making these control system measurements that makes one the best choice for any given application.

If these measurement considerations are taken into account during the design stage of the control system, then connecting the analyzer can be as simple as connecting its input channels to pre-configured test points.

Control System Measurement Accessories

HP has three control system accessories that are designed to be compatible with most control systems and HP's network and dynamic signal analyzers. One device that may be the most versatile in the frequency range of DC to 1 MHz, is the HP 35280A active summing junction as shown in figure 18.

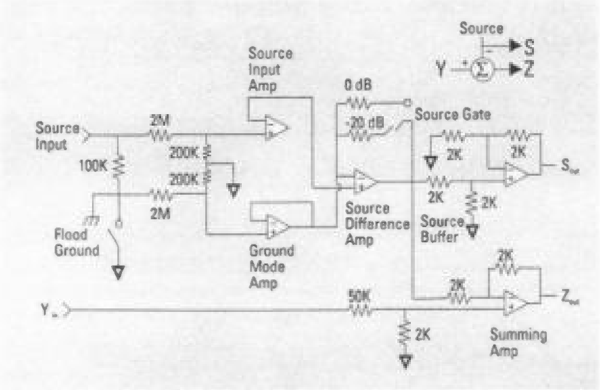


Figure 18: HP 35280A summing junction

This device offers a +/- 10 volt signal range and allows the summing junction signal to float to ±42 Vpk.

A transformer-based device such as the HP 35282A, schematically shown in figure 19, is more appropriate for applications needing a higher voltage rating (600 Vpk secondary float voltage) or simply a passive device. Its frequency range is 30 Hz to 200 kHz.

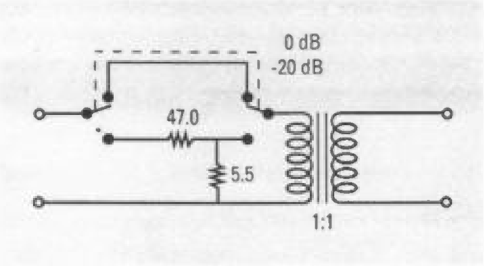


Figure 19. HP 35282A 30 Hz to 200 kHz Transformer

The HP 35281A "clip-on" transformer in figure 20, has the highest frequency performance of 300 Hz to 10 Mhz. It offers the convenience of a simple "clip-on" probe for signal injection.

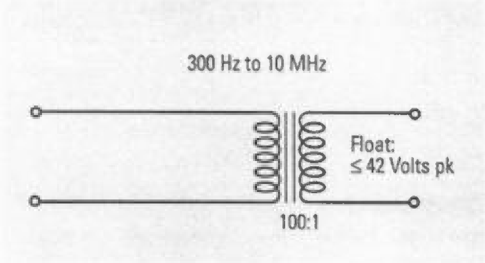


Figure 20: HP 35281A 300 Hz to 10 MHz transformer

More detailed specifications for these three accessories can be found in Appendix A.

Section 4

Typical Measurements

To demonstrate these methods and devices we will make a series of measurements on a control system which has the following analytical description for the open-loop gain, GH:

$$\frac{7x10^7}{s^3 + 972s^2 + 78944s + 1018368}$$

The gain margin for this system is analytically 0.68 dB at a frequency of 281.0 Hz and the phase margin is 1.3 degrees at a frequency of 270.25 Hz. The gain at dc (0 Hz) is 36 dB. This open-loop gain function is plotted in figure 21 using the synthesis capability of the HP 3563A control system analyzer. This figure plots the analytical function using 801 lines from dc to 1 kHz, with a corresponding frequency resolution of 1.25 Hz. (Since the data in this application note was taken with more than one HP analyzer, all the data was plotted using the Standard Data Format (SDF) utility "Viewdata." This SDF data format is a standard feature of all current Hewlett-Packard dynamic signal analyzers and allows these plots to be imported directly into many word processor programs, for example, Microsoft®'s Word for Windows and Lotus'® AMI Pro.)

With this frequency resolution the observed phase margin using the marker function is 1.4 degrees at a frequency of 269 Hz.

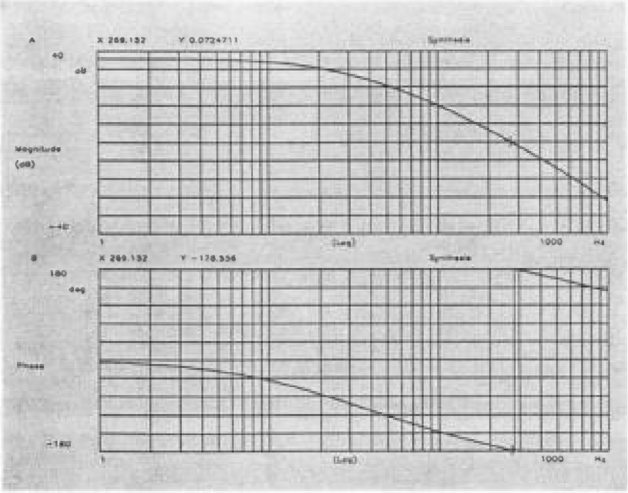


Figure 21: Analytical open-loop gain

As discussed earlier, there are a number of measurement locations around the control-system loop from which the open-loop gain function can be estimated. In HP application note 243-5, table 1 presents 18 ways to accomplish this; this information is reproduced in table 1 of this application note. Of these methods, eight of them use a swept-sine excitation with the SFT analysis method. The other ten methods use a broadband excitation and the FFT analysis method.

SFT and Swept-Sine Excitation Using the HP 35280A Summing Junction

We will start our practical look at these measurements by using the SFT method and one of the more familiar connection methods, Y/S. To be consistent with AN 243-5, this measurement will be referred to as YSS since we will use the SFT (indicated by the second letter in YSS). The HP 3563A will be the instrument used to make the actual measurements. The summing junction shown inside the loop in figure 22 is the HP 35280A.

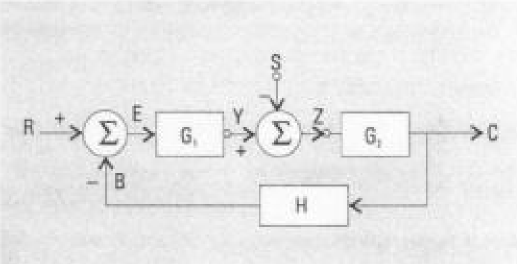


Figure 22. General internal excitation case

To set up the analyzer perform the selection tasks outlined below in Table 2:

Table 2:

MEAS MODE	Swept Sine Log Resolution
SELECT MEAS	Frequency Response
AVG	Number of Avg = 1 Auto Integrate
FREQ	Start Frequency Stop Frequency Resolution: 66pts/dec
SOURCE	Level 50 mv
RANGE	Auto up and down
COUPLING	DC and Floating

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Table 1: Measurement Method Summary

External Signal Injection Methods

1*	B+E	1	$BE^*/\lvert E\rvert^2$	(BE*+BB*)/(EB*+EE*)	No bias, minimum variance
2*	BSE	3	BS*/ES*	T	No bias, low variance
3	B/E	12	$BE^*/\lvert E\rvert^2$	T	Potential bias, minimum variance
4*	ESA	4	ES*/AS*	(1-T)/T	Low bias, low variance
5*	E/A	2	$EA^*/\lvert A\rvert^2$	(1-T)/T	Low bias, minimum variance
6*	ESS	5	$ES^*/\lvert S\rvert^2$	-(1-T)/T	Low bias, low variance
7*	E/S	5	$ES^*/\lvert S\rvert^2$	-(1-T)/T	Indentical to number 6
8	BSA	9	BS*/AS*	T(1-T)	Potential bias, high variance
9	B/A	8	$BA^*/\lvert A\rvert^2$	T(1-T)	Potential bias, high variance
10	BSS	11	$BS^*/\lvert S\rvert^2$	-T(1-T)	Potential bias, high variance
11	B/S	11	$BS^*/\lvert S\rvert^2$	-T(1-T)	Identical to number 10

Internal Signal Injection Methods

12*	Z/S	7	$ZS^*/\lvert S\rvert^2$	-(1+T)/T	Low bias, medium variance
13*	ZSS	7	$ZS^*/\lvert S\rvert^2$	-(1+T)/T	Indentical to number 12
14*	Y-Z	6	$YZ^*\lvert Z\rvert^2$	(YZ*-YY*)/(ZY*-ZZ*)	Low bias, medium variance
15*	YSZ	6	YS*/ZS*	-T	Similar to number 14
16	Y/Z	13	$YZ^*/\lvert Z\rvert^2$	-T	Bias, medium variance
17	YSS	10	$YS^*/\lvert S\rvert^2$	T/(1-T)	Potential bias, high variance
18	Y/S	10	$YS^*/\lvert S\rvert^2$	T/(1-T)	Identical 17

Table 1 reprints the table found in AN 243-5 summarizing the 18 "measurement connection" methods to compute the open-loop gain function.

The asterisks in the first column of Table 1 denotes that these methods are good for large loop-gain magnitudes and the maximum measurable loop-gain magnitude is limited by the ratio of excitation signal amplitude to analyzer noise amplitude.

The HP 3563A analyzer has some useful features for making good measurements in the swept-sine mode. One of the more significant contributions is the dynamic auto-ranging capability. By setting "auto range up & down" the analyzer continuously adjusts the input range of both channels to an optimum value at each point of a sweep. In this way the dynamic range of a measurement can exceed 140 dB.

Setting the source level requires some care on the user's part. In many control systems the loop gains are quite large and at either the frequency(s) where the gain or phase margins are minimum or at resonant frequencies, the signal levels within the control loop or the controlled variable can become large. It is usually worthwhile to determine the optimum level of the source driving signal before making detailed measurements. A value that achieves good signal-to-noise ratios without saturating a gain stage or exceeding some response level criteria should be determined. A practical approach for determining this signal level involves starting at a low source level and then manually sweeping through the critical frequencies while observing the signal levels (both electrically and mechanically) at several locations around the control loop.

If the device under test has high Q resonances (lightly damped poles), another helpful feature of the HP 3563A is "auto resolution". The sweep is adjusted to obtain many points as the resonances are traversed and fewer points (coarser resolution) outside these regions. This results in a measurement data set with non-evenly spaced frequency points. The curve fitter within this product can accommodate this uneven frequency spacing as well as the more common linear or log frequency spacing. This capability minimizes the amount of time required to make the measurement by sweeping quickly through low Q (highly damped) regions and slowly through high Q resonances.

Having made the physical connection between the analyzer source output and the "S" input on the HP 35280A, connect the buffered "S" signal from the HP 35280A to channel 1 and the "Y" signal to channel 2 of the analyzer.

Making a YSS measurement results in the estimate of the open-loop gain function, GH, shown in figure 23. The total measurement time was a little over 6 minutes.

To obtain this estimate from the measured data YSS one needs to perform the waveform math operation:

$$\mathrm{GH} = \frac{T}{1-T}$$

This has been done to the data shown in figure 23.

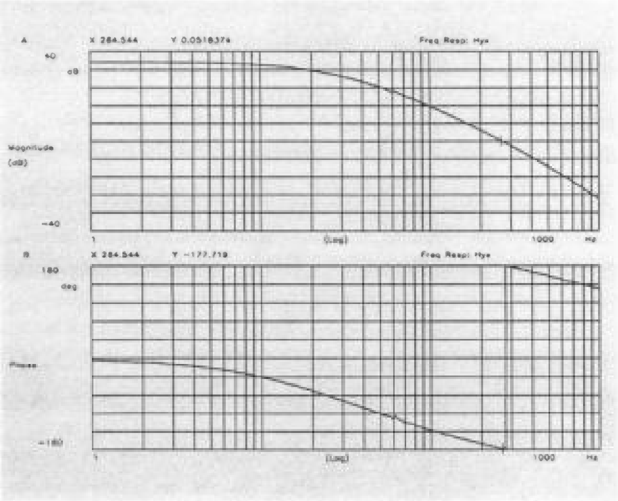


Figure 23. Open-loop gain, GH, from a YSS measurement

From this graph one can determine the measured estimate of the open loop-gain by simply using the x-marker function built into the analyzer. This measured estimate of the phase margin is:

Phase Margin = 2.281 degrees @ 264.54 Hz.

Using just this x-marker, the frequency line closest to 0 dB resides at 264.54 Hz with a value of +0.0518374 dB.

To improve this estimate, the analyzer has a special marker function that interpolates the data and determines a frequency and phase value where the gain is actually 0 dB (as well as, a value of gain where the phase is -180 degrees). Using this feature we obtain the following estimate of the gain and phase margins:

Gain Margin = 0.627 dB @ 277.4 Hz
Phase Margin = 2.04 degrees @ 265.6 Hz.

The measurement that best resembles the anticipated answer based upon the analytical function was obtained from the YSZ swept-sine method. This measurement is shown in figure 24 below. The main disadvantage to this method is the longer measurement time, it requires 12 minutes to complete. Most of the additional time arises from the auto-ranging of the input channels and using the auto-integrate feature which integrates until the variance reaches a predetermined level (in this case 5%).

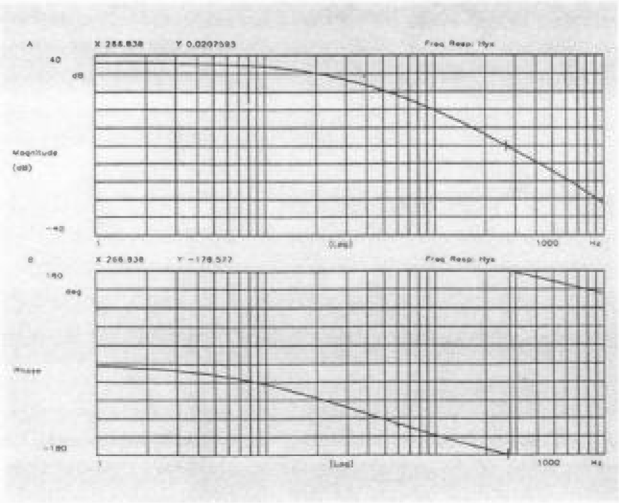


Figure 24: Open-loop Gain Function from YSZ Measurement.

FFT and Burst Random Excitation

Instead of using swept-sine excitation and the SFT, the next measurement is made using the narrowband FFT mode and burst random excitation.

Burst random excitation is basically a burst of random noise that starts at the same time that the analog-to-digital converter (ADC) begins acquiring a "T" second data record. In this measurement "T" is comprised of 2048 samples of ADC data from both channels. To avoid "leakage" errors when these records are Fourier transformed we will adjust the duration of this random excitation to be less than "T". The duration in this experiment was 70%. The 30% "off" time allows the system response to decay to near zero before the end of the data record, thus avoiding leakage errors when Fourier transformed.

The correct choice for the window function in this measurement is a rectangular (uniform) window (i.e. no window function applied). Figure 25 shows the time histories of both channel 1 and channel 2 (S and Y).

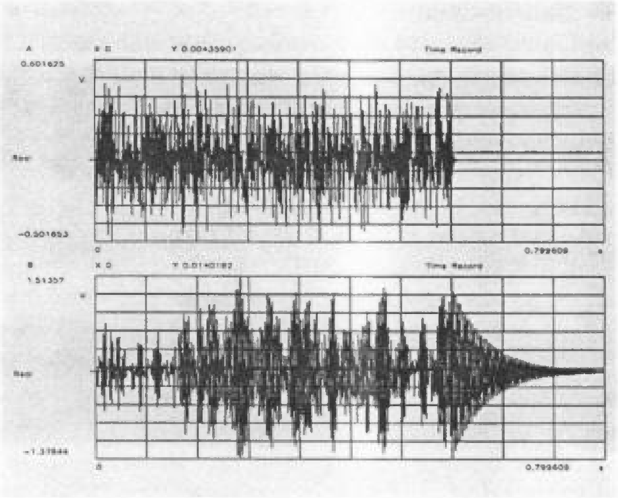


Figure 25: Upper Trace is S and lower is Y

In the upper trace of figure 25, the random excitation begins at T=0 and continues for 70% of the time record. The lower trace shows the response at Y to this excitation and also the response decay once the excitation goes to zero. For the assumption of negligible leakage the response should decay to almost zero before the end of the record. If the decay time takes longer than 30 to 40% of the record then the time record length, T, of the measurement should be increased. This can be accomplished either by using the zoom mode to increase the frequency resolution (smaller Δf) thereby increasing the record length T or (in some analyzers) by increasing the transform (blocksize) size. If the "on" time of the excitation gets shorter than about 50% then there's a potential signal-to-noise problem because of the short excitation times. The analyzer setup for this FFT mode is outlined in the selection tasks in table 3.

Table 3:

MEAS MODE	Linear Resolution
SELECT MEAS	Frequency Response
WINDOW	Uniform
AVG	Number of Avg = 10 Stable (Mean)
FREQ	Span 1 kHz Zero Start
SOURCE	Type: Burst Random Level: 0.8 V
TRIGGER	Source Trigger
RANGE	Manual or Auto Up
COUPLING	DC and Floating

Y/S Measurement

The resulting open-loop frequency response measurement from this method is shown in figure 26. This measurement using 10 averages, required a total measurement time of approximately 10 seconds. More averages should have been taken to reduce the variance, but a reasonable estimate of the gain and phase margins were obtained in 10 seconds compared to 6 minutes using SFT.

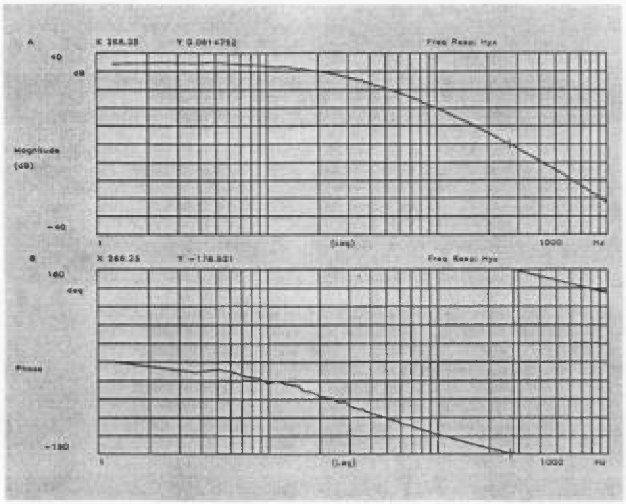


Figure 26: Open-loop gain measurement GH obtained from Y/S

The speed advantage of the narrowband FFT mode over the SFT mode is not always achievable in systems where there's poor signal-to-noise ratios or when the system contains non-linearities. This particular test device had very little external contaminating noise, so the FFT mode typically will be much faster, particularly if the measured frequency response function contains a high Q resonance. The actual Y signal spectrum contains this high Q resonant behavior and is similiar to what is shown in figure 30.

This speed advantage is the result of the SFT requiring a sweep slow enough to allow the device under test and/or the tracking filter in the analyzer to always reach a steady state result. In the FFT mode, since it is equivalent to an 800-line parallel-filter analyzer we only have to wait for this settling time once as opposed to 800 times if we were to produce a 800 frequency result using the SFT mode.

When we are dealing with signals that have a very poor signal-to-noise ratio, the number of averages required in the FFT mode becomes large and this speed advantage over the SFT largely disappears.

Z/S Measurement

A measurement using Z/S can also be made which is ranked one step better in AN 243-5. Once Z/S is obtained the math operation needed to convert it to an estimate of the open loop gain is:

$$GH = -\frac{1+T}{T}$$

Figure 27 represents the result of this measurement, also made with burst random excitation and only 10 averages. The measured estimates of the gain and phase margins are (using the special marker function):

Gain Margin = -0.706 @ 278.1 Hz
Phase Margin = 1.36 degree @ 267.2 Hz

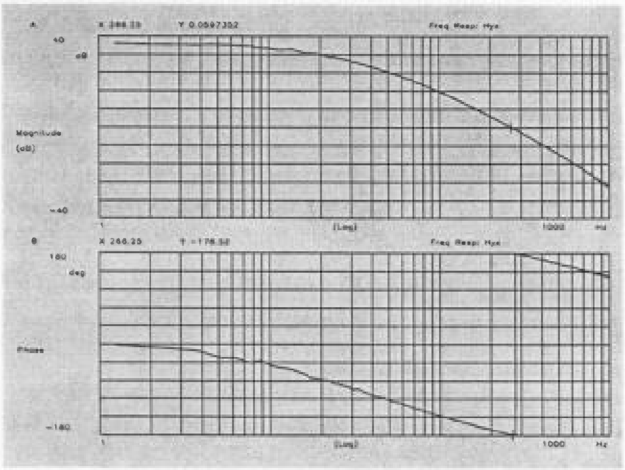


Figure 27: Open loop gain function GH obtained from Z/S

The methods YSS, Y/S and Z/S are typical of the measurements made in the recent past. One contribution of Application Note 243-5 has been to describe analytically the measurement errors of traditional methods as well as some new techniques. AN 243-5 points out that the traditional measurement locations are not the best locations based on the rankings presented in that application note. The reason for these rankings are the bias errors that can arise from the various noise terms in the measurements and the variances.

The method that has probably been the most popular has been Y/Z and in AN 243-5 it has the worst ranking. Using the same burst random excitation with 10 averages we obtain the measurement in figure 28. With Y/Z there is no waveform math operation needed other than a simple multiplication by -1. Y/Z is a direct estimate of the open-loop gain function which probably has been the reason for its popularity.

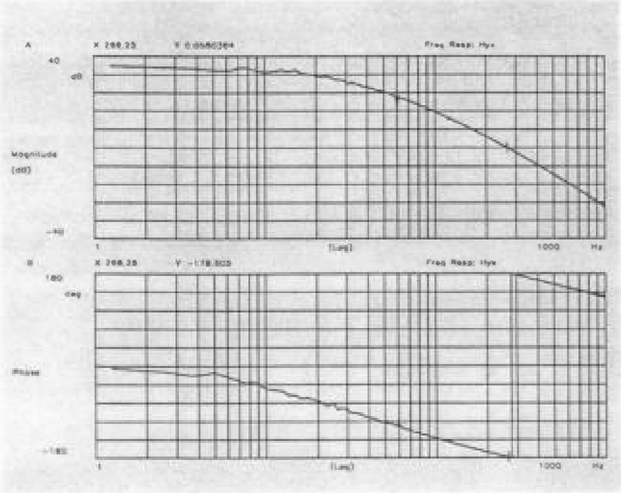


Figure 28: Open-loop gain GH obtained from Y/Z measurement

The variance appears higher at lower frequencies which can be expected since this method is more susceptible to noise both in terms of variance and bias.

Section 5

Transformer Injection Devices

FFT Mode and Burst Random Excitation

Instead of making the measurements on the example device using the HP 35280A summing junction, we could have used the HP 35282A signal injection transformer. The measurement in figure 29 is an identical measurement setup to that shown in table 3, using the FFT mode and a burst random excitation. The measurement connection method is Y/Z, and the number of averages taken is 50.

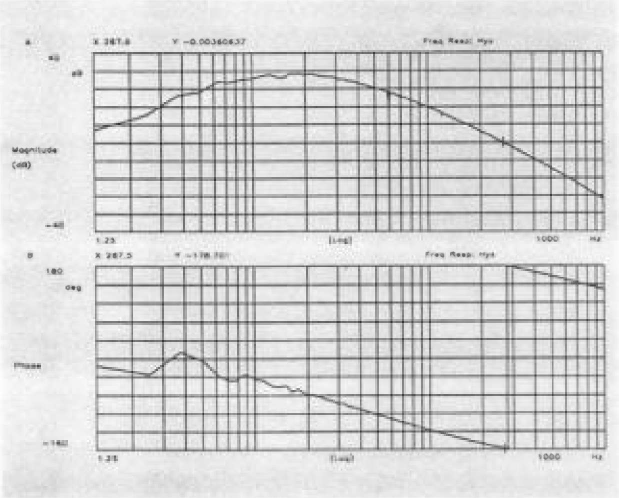


Figure 29: Y/Z Measurement Using HP 35282A Signal Injection Transformer

The measurement produced a reasonably good estimate of the gain and phase margins for this device. There is a problem of not being able to couple the excitation signal below about 30 Hz. The lower frequency excitation roll-off causes the signal levels to fall below the dynamic range of the analyzer when it is in the broadband FFT mode. Figure 30 shows the power spectrum of the measured input and output signals. Notice that in the upper trace of the power spectrum of the Z signal, that the dynamic range of this spectrum is about 85 dB. The small signals in the low-frequency region due to the high-loop gain (at these frequencies) and the lack of adequate excitation at the lower frequencies due to the transformer cutoff, combine to make this measurement only useful above approximately 30 Hz.

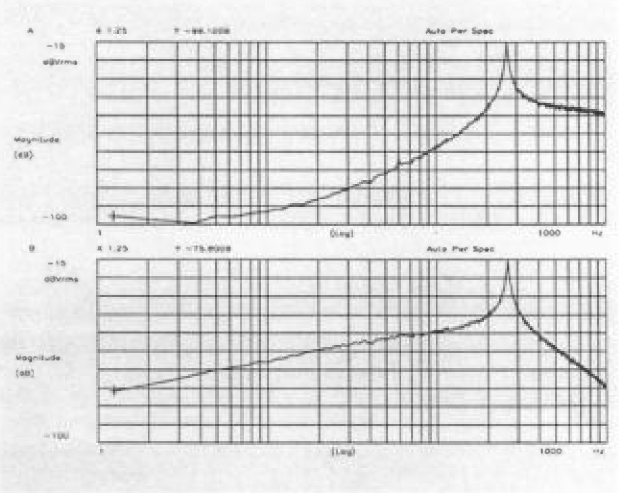


Figure 30: Input and Output Power Spectrums of Z and Y

SFT Mode and Swept-Sine Excitation

Making a new measurement using the swept-sine mode of the analyzer allows the dynamic range to be extended from 80 dB to over 140 dB. Figure 31 shows this swept-sine measurement.

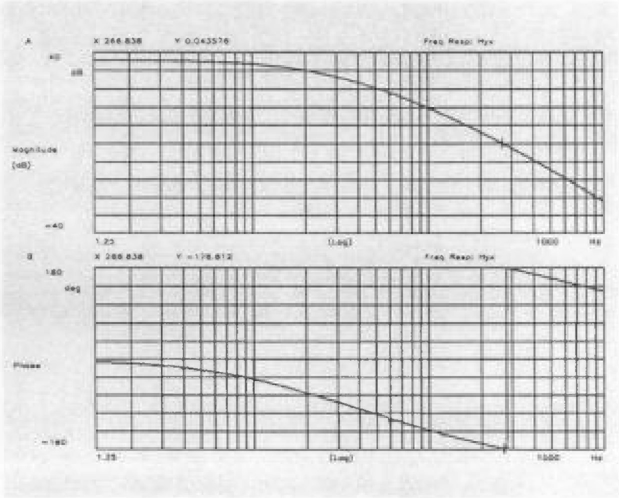


Figure 31: YSZ measurement using the HP 35282A transformer

Because of this increased dynamic range which arises from allowing the range setting to be optimized at each frequency as the sweep "steps" along, the measurement can almost extend down to DC (zero frequency). Swept sine is most likely the best method to use, especially with transformer injection devices, if the additional measurement time can be tolerated.

Section 6

Disk-Drive Servo Loop

The device under test in the previous measurements allows for a fairly straight forward measurement and most methods yield good to very acceptable results. Other devices can create less than ideal measurement environments, for example, the head positioning servo of a disk drive. Because of the mechanical components and the common problems associated with these devices such as "run out", the signals to be measured are very noisy. Also, the rotational speed of 3600 rpm causes a considerable amount of 60 Hz noise.

To make these measurements, the HP 35280A summing junction was inserted inside the loop just before the power amplifiers driving the actuator.

The first measurement made was Y/Z, mainly because it is fast and easy to do. Because of the high amount of background noise, the excitation signal chosen was random noise. A Hann window was selected to address the leakage that is inherent in these signals. Figure 32 shows the measurement Y/Z multiplied by -1 to obtain the open-loop response. The signal-to-noise ratios are only about 1:1 as observed in the time domain. Note all of the anomalies at multiples of 60 Hz. This measurement contained 1000 averages at 800 lines of resolution and required approximately 8.5 minutes to complete in the fast averaging mode (display update after averaging complete). In an attempt to reduce the 60 Hz contamination, the number of averages was chosen to be 1000.

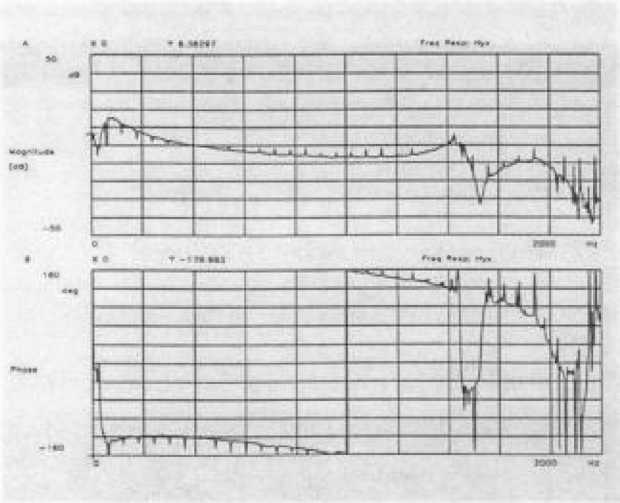


Figure 32: Y/Z measurement of disk servo with 1000 averages

The results of a Y/S measurement ranked better in AN 243-5 is shown in figure 33. This figure has had the waveform math operation T/(1-T) applied to estimate the open-loop gain function, GH. This measurement was made with 700 averages and required approximately 6 minutes. A random noise excitation was used with a Hann window. Originally, this measurement was set up for 1000 averages based upon the results obtained using Y/Z, but the averaging was stopped after observing reasonable results after 700 averages.

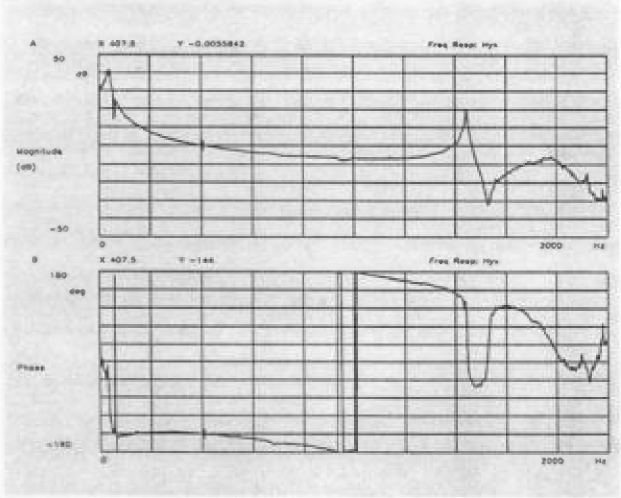


Figure 33. Disk drive open-loop measurement derived from Y/S

Using the swept-sine approach, a YSZ measurement of the disk drive yielded the measurement shown in figure 34. This measurement required approximately 12 minutes to complete with a linear sweep and a resolution of 2.5 Hz.

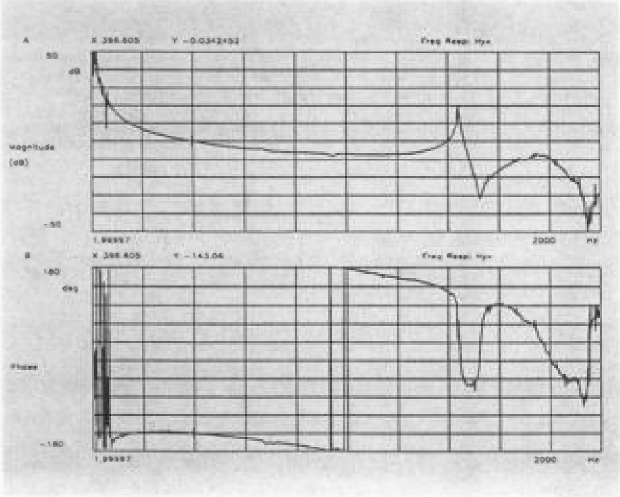


Figure 34: YSZ measurement using linear sweep

Using the New Method Y-Z from AN 243-5

Among the internal (inside the loop) excitation methods, the best method from the rankings of AN 243-5 is derived from the components (auto and cross power spectrums) of the lowest ranked method. Figure 35 shows the input power spectrum of Z (ZZ*) in the upper trace and the output power spectrum of Y (YY*) in the lower trace. Figure 36 shows the cross power spectrum YZ* with the magnitude in the upper trace and phase in the lower trace.

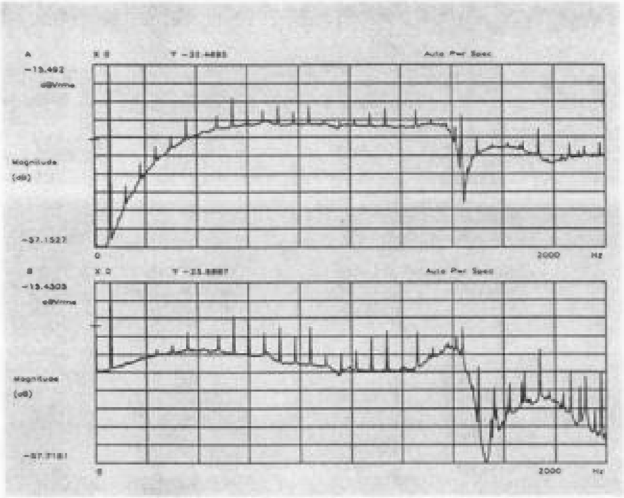


Figure 35: Power spectrum $|Z|^2$ or (ZZ*) in upper trace and $|Y|^2$ or (YY*) in lower trace

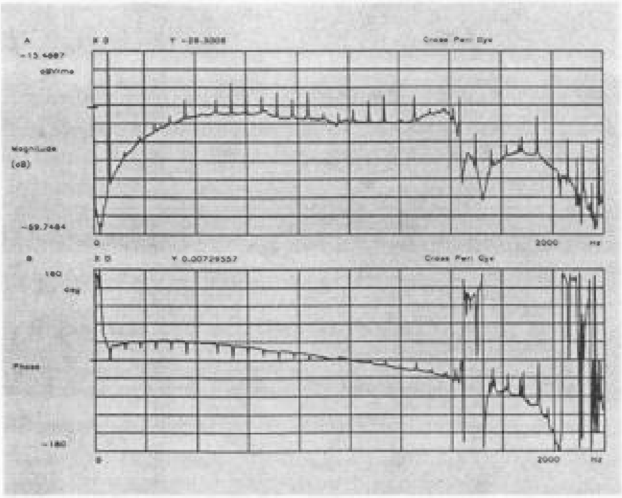


Figure 36: Cross power spectrum YZ*

Note that in figure 36 the phase spectrum needs to be multiplied by -1 to allow comparison to the phase spectrum of the open-loop gain function GH shown in previous figures.

The best method is denoted Y - Z and is computed from the auto and cross spectrums of Y and Z as follows:

$$\frac{YZ^* - YY^*}{[YZ^*]^* - ZZ^*}$$

The results of this calculation are shown in figure 37. This plot shows that the Y-Z method removes the 60 Hz harmonics that were seen in the direct Y/Z method. The bias has also been removed which results in a better estimate of the resonant peak around 1450 Hz. The low frequency gain estimate is also better, although not as good as the estimate obtained from the swept-sine measurement YSZ.

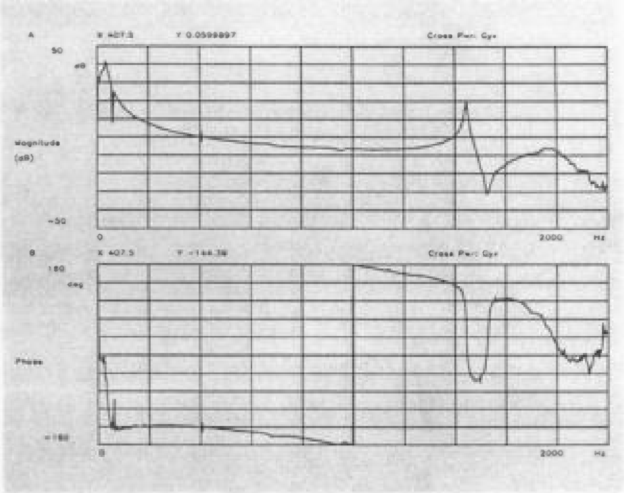


Figure 37: Open-loop gain function, GH, computed from Y-Z

FFT Mode and Burst Chirp Excitation

Since the ratio of the peak-to-rms value of a random noise excitation is high compared to a swept-sine chirp excitation, there is some advantage to using the sine chirp in cases where the device under test has a response limit. An example is a control surface on an airplane, where there are mechanical stops limiting travel to several degrees in either direction. In this case, the use of a swept-sine chirp would allow a better signal-to-noise ratio to be obtained since the peak-to-rms ratio is lower for the swept sine (assuming the control surface amplitude is driven in both cases to near the mechanical stops).

Using the same measurement setup as table 3 on page 11, we need to only change the analyzer to output a burst chirp (burst swept sine) with source trigger, and select a uniform window. The analyzer used in this experiment is the HP 35665A dynamic signal analyzer, since we want to demonstrate vector averaging. Figure 38 shows the time waveforms for both the Z and Y signals using this setup. In this measurement the burst length of fast sine sweeps is 70% of the block time. The sweep rate is very fast since the sweep covers a span of 1600 Hz in 70% of the block time. The block time, T, is 0.5 second since this analyzer has been setup for 800 lines and 1600-Hz span. The only way to process these extremely fast sweep rates is with the fast fourier transform (FFT) mode since these sweep rates do not allow for a steady state response of the device under test.

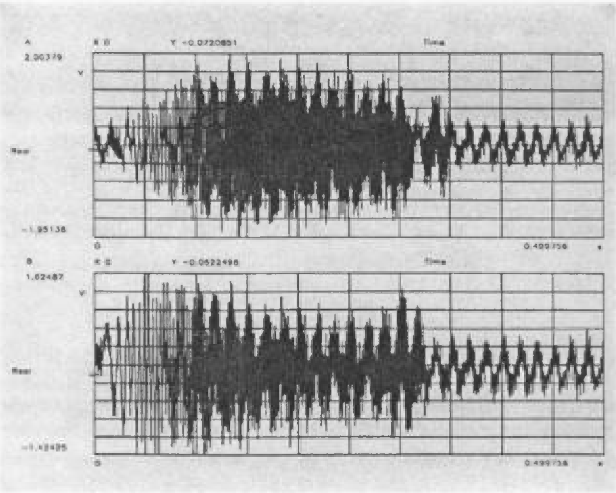


Figure 38: Burstc chirp response signals for Z in the upper trace and Y in the lower

The last 25% of the responses in figure 38 depicts the fairly high background noise levels. The signal-to-noise ratios in these measurements are approximately 3:1. We can use vector averaging (equivalent to time domain averaging) in these measurements since we repetitively output the same sweep signal synchronized to the start of each time record. This process should average out the signals that are not correlated to the excitation sweep. Figure 39 shows the results of these measurements after 255 averages. It turns out that the direct measurement of Y/Z is identical to Y-Z, when vector averaging is used.

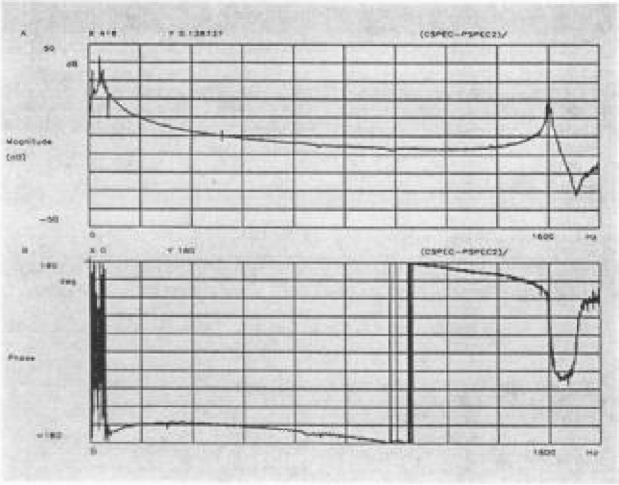


Figure 39: Y-Z measurement with a burst sine chirp and vector averaging

If we repeat the measurement using RMS averaging instead of vector averaging, we again see a difference between Y/Z and Y-Z as shown in figure 40. Y/Z is in the upper trace and Y-Z is in the lower.

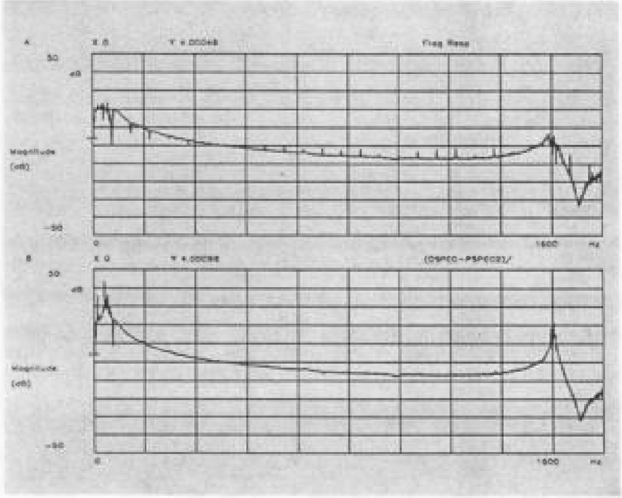


Figure 40: Y/Z and Y-Z measurements with a burst chirp and RMS averaging

Switching Mode Power Supply

Using these control system measurement techniques on a switching mode power supply starts with deciding where to place the signal injection device and addressing some aspects of these choices. In a multi-loop design, it is important that the signal be injected at a point where all loops are excited.

In the following example, one choice for the signal injection device is the HP 35280A summing junction in place of R2 in figure 42.

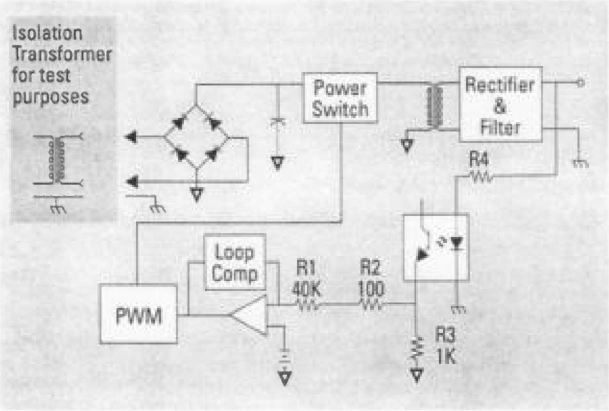


Figure 42: Switching mode power supply

The primary side is often the best place for signal injection, but it typically floats 1/2 the line voltage. To allow grounding of the primary common, a line isolation transformer should be used. This can be accomplished by using a medical isolation transformer (leakage < 50 microamps) such as the Magnetek Triad # N-90MD rated at 115 V and 250 VA.

Figure 43 shows the open-loop gain function, GH, obtained from a -Y/Z measurement. This measurement most likely contains a slight gain error since the input impedance of the Y input of the summing junction is not identical to normal load impedance of the operating circuit. The swept-sine measurement was made with the HP 35665A. The anomaly at 120 Hz is due to the feedback attempting to correct for rectifier ripple.

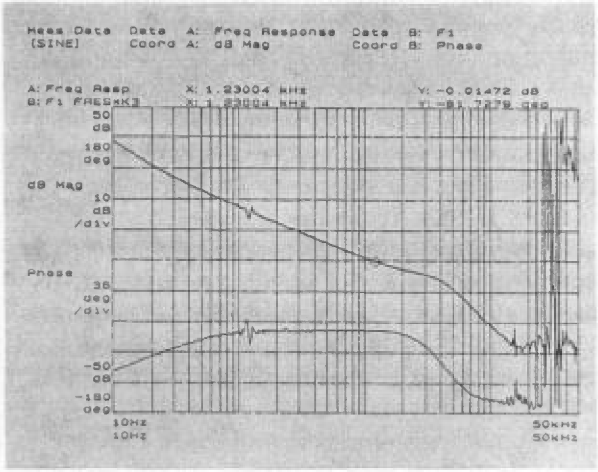


Figure 43: Open -loop gain function, GH, obtained from -Y/Z measurement

Another way of connecting a signal injection device to this circuit is with the HP 35282A transformer which can simply be connected across the 100 ohm resistor R2. The Y and Z measurement connections are shown in figure 44.

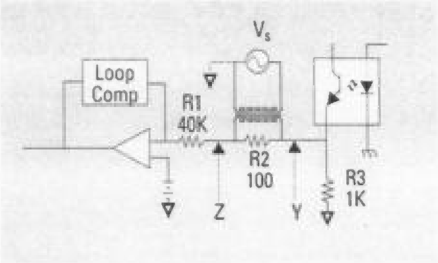


Figure 44: Signal injection with the HP 35282A transformer

The -Y/Z measurement with this setup is shown in figure 45. This measurement obtains an estimate of the open-loop gain, GH, that degrades above 5 kHz because of the increase in the impedance of the emitter follower within the optical isolator.

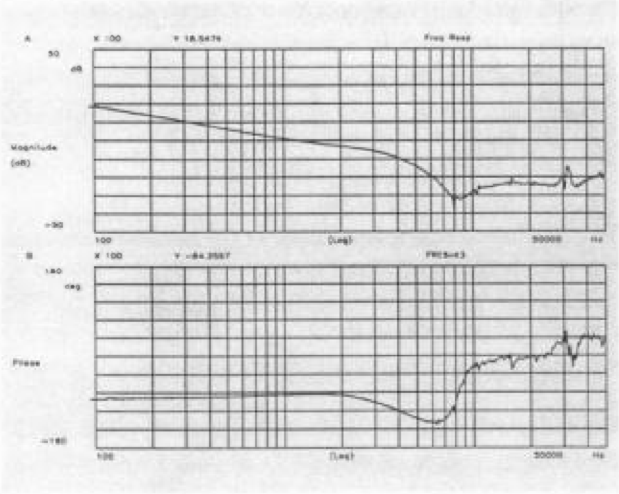


Figure 45 : Open-loop gain, GH, obtained using the HP 35282A transformer

A more creative way of connecting a signal injection device to this circuit is to use the HP 35280A summing junction as an "active transformer" as shown in figure 46. This connection allows the source to float and has a minimum affect on the impedance within the circuit. The resulting measurement of the gain function, GH, is shown in figure 47.

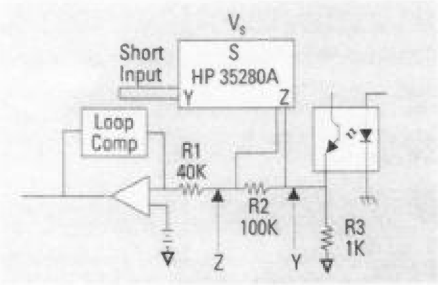


Figure 46: Using the HP 35280A as an active transformer

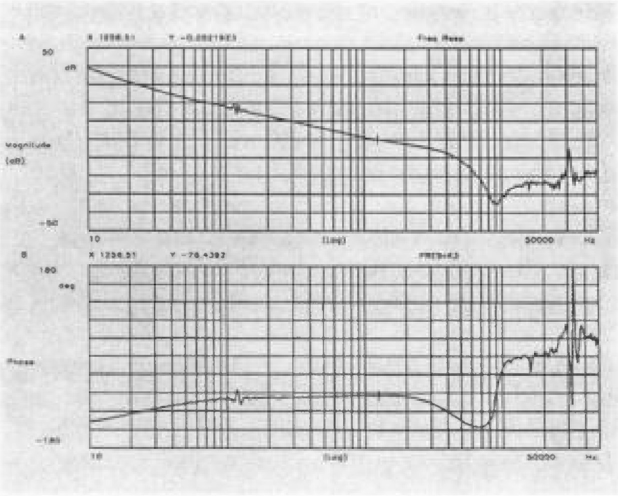


Figure 47: Open-loop gain function GH obtained from the active transformer

Figure 48 is the same measurement as was setup in figure 46 using the HP 3577A instead of the HP 35665A.

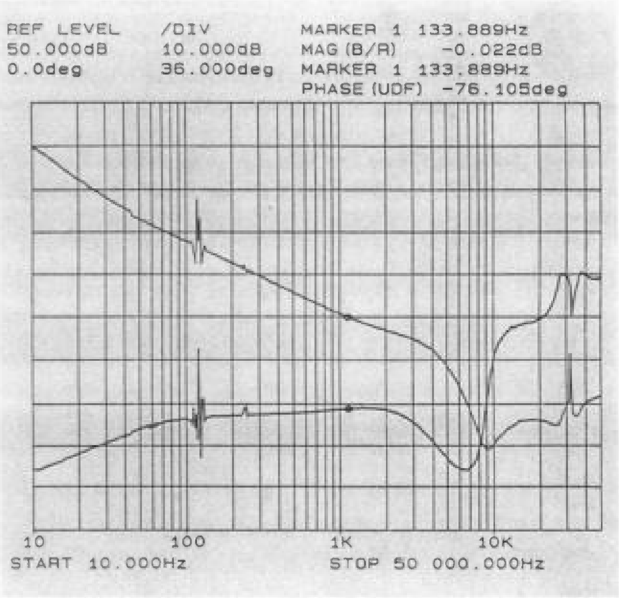


Figure 48: Measurement obtained with an HP 3577B

Section 8

Summary

There are a number of methods which are available to make effective measurements of control system open-loop gain functions. The choices include the use of a summing junction or transformer as the signal injection device, the choice of signal excitation type (sine versus broadband and what type of broadband excitation), the calculation method (Y/Z, Y-Z, Y/S, etc.) and the capabilities of the analyzer used. All significantly impact the quality of the measurements obtainable in any given situation.

Table 4 attempts to summarize the choices and the significant advantages/disadvantages of the various choices available in terms of the excitation types. This table is only a guide and should not be interpreted as absolute; there are always exceptions to any attempt to generalize.

Hewlett-Packard has a number of analyzers with overlapping capabilities that address these measurement needs:

HP 3562A dynamic signal analyzer
HP 3563A control systems analyzer
(Includes digital inputs/outputs)
HP 35665A dynamic signal analyzer
HP 3566A/3567A PC-based multichannel
spectrum/network analyzer
HP 3577B network analyzer
HP35670A dynamic signal analyzer

Consult with your local Hewlett-Packard sales representative for information on the newest analyzer, accessory or software package addressing these measurement needs.

Table 4:

SFT (Swept Fourier Transform) Analyzer Mode	FFT (Fast Fourier Transform) Analyzer Mode							
	Non-Periodic	Periodic in Analyzer Window			Transient within Analyzer Window			
Sine Steady State	True Random Noise	Sine Chirp (Fast Sine Sweep)	Pseudo Random Noise	Periodic Random Noise	Impact or Delta-Function	Burst Sine Chirp	Burst Random Noise	
---	NO	Yes	Yes	Yes	Yes	Yes	Yes	Minimum Leakage
Very High	Fair	High	Fair	Fair	Low	High	Fair	Signal-to-Noise Ratio
Poor	Good	Good	Good	Fair	Good	Very Good	Very Good	Measurement Speed
Yes	No	Yes	No	No	No	Yes	No	Characterize Nonlinearity
Fair	Good	Good	Good	Fair	Fair	Good	Good	Setup Ease of Use
Very High	Fair	High	Fair	Fair	Poor	High	Fair	RMS to Peak Ratio
Yes	No	Yes	No	Yes	Yes	Yes	No	Vector Averaging Applicable
Very High	Fair	Good	Fair	Fair	Fair	Good	Fair	Measurement Dynmaic Range
---	Hann	Rect	Rect	Rect	Rect	Rect	Rect	Normal Window Function

References:

[1] Control System Development Using Dynamic Signal Analyzers, Hewlett-Packard Application Note 243-2.

[2] Control System Loop-Gain Measurements, Hewlett-Packard Application Note 243-5.

[3] Control System Measurement Techniques and Coherence Calculations, Hewlett-Packard Technical Notes. Publication Numbers 5959-5760 and 5959-5761. Printed as a single document.

[4] Measuring Nonlinear Distortion Using the HP 3562A Dynamic Signal Analyzer, Hewlett-Packard Product Note 3562A-4.

[5] Measuring the Open-Loop Frequency Response of the Phased-Locked Loop, Hewlett-Packard Product Note HP 3562A/3563A-1.

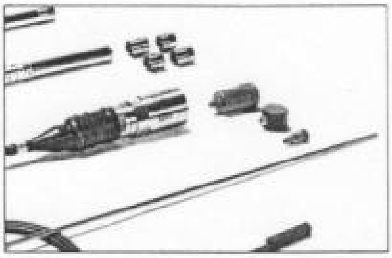
[6] Measuring the Open-Loop Frequency Response of the Phase-Locked Loop, Hewlett-Packard Product Note HP 3577B-1.

Appendix A

HP Accessories For Control System Measurements

HP 35280A, HP 35281A & HP 35282A Accessory Specifications:

All of these accessories let you inject signals into your feedback control systems, such as disk drives, power supplies, AGC, and PLL circuits. You can characterize loop gain using E/A, Y/S, Y/Z, or other measurement techniques. These accessories are compatible with a wide range of network and dynamic signal analyzers, including the HP 35670A, HP 35665A, HP 3562A, HP 3563A, HP 3566A/67A, and HP 3577B.



Physical Sensors Catalog

Applications that require measurement of physical quantities like acceleration, velocity, acoustic sound pressure and temperatures might benefit from the Hewlett-Packard Physical Sensors Catalog (5952-2996). This catalog contains several of the more popular general purpose transducers available. It also contains instrumented hammer kits for Modal Testing.

HP 35280A Summing Junction

The HP 35280A summing junction allows you to float the injected signal source from ground up to ±42V. You can also attenuate the source gain by -20 dB. Includes the required line power module.

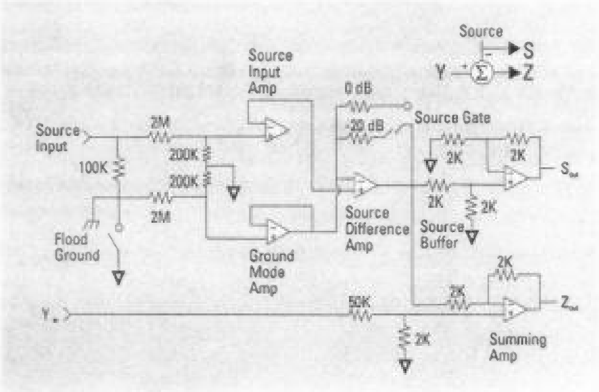


Figure A-1. Simplified block diagram of HP 35280A summing junction

Specifications:

Frequency Range: dc-1MHz (<10° phase shift)

Voltage Range: ±10 Vpk

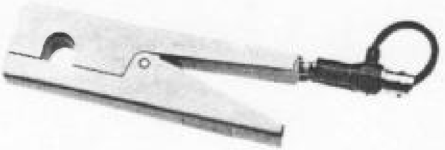
Maximum Float Voltage: ±42 Vpk

Gain: 1 (0 dB)

Input Impedance: 100 kΩ

Output Impedance: <15 Ω

Source attenuator: 0 dB or -20 dB



HP 35281A Clip-on Transformer

The HP 35281A clip-on transformer lets you inject a test signal into higher frequency servo control loops and other feedback loops, such as AGC circuits and switch mode power supplies. This transformer lets you clip-on to an existing lead for injecting a test signal without physically breaking the circuit.

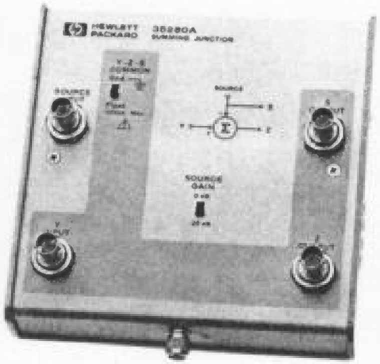
Specifications

Frequency Range: 300 Hz-10 MHz (roll-off <6 dB) Typical

Max. Source Primary Voltage for harmonics <-40 dBc
300 Hz: 0.1 Vpk (typical)
5 kHz: 5 Vpk (typical)

Maximum Secondary Float Voltage: ±42 Vpk

Primary to Secondary Turns Ratio: 100 to 1 (-40 dB ±1.5 dB at 100 kHz)



HP 35282A Signal Injection Transformer

The HP 35282A signal injection transformer allows you to inject signals into your control loops which have large offsets from ground. This device allows offsets of up to ±600 Vpk, and provides signal attenuation of 0 or -20 dB.

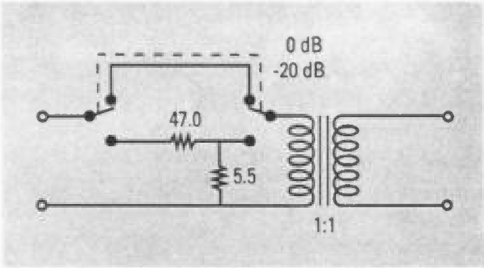


Figure A-2. Simplified block diagram of HP 35282A Signal Injection Transformer

Specifications

Frequency Range: 30 Hz- 200 kHz (roll-off <6 dB)

Max. Source Primary Voltage for harmonics <-40 dBc
30 Hz: 0.05 Vpk (typical)
1 kHz: 5 Vpk (typical)

Maximum Secondary Float Voltage: ± 600 Vpk

Primary to Secondary Turns Ratio: 1 to 1 (0 dB ±0.1 dB at 1 kHz)

Source Attenuator: 0 dB or -20 dB

Primary to Secondary Common Mode Response (at 1 kHz): < -100 dB