Extending Vector Signal Analysis to 26.5 GHz with 20 MHz Information Bandwidth

Product Note 89400-13

Figure 1.
HP 89410A vector signal analyzer and HP 71910A wideband surveillance receiver together form a wideband vector signal analyzer system.

The HP 89400 series vector signal analyzers provide unmatched signal analysis capabilities from traditional spectrum analysis to transient analysis, analog and digital demodulation, high-speed spectrum monitoring, phase noise analysis and more. They are used in applications as wide ranging as surveillance, signal monitoring, digital communications, radar signal analysis and underwater acoustics.

For some applications, the vector signal analyzer's information bandwidth and frequency coverage has been a limitation. Instruments such as the HP 89441A are limited to frequencies below 2.65 GHz and information bandwidths of 7 MHz. This precludes the analysis of many

spread spectrum, radar and satellite signals which typically occupy more than 7 MHz bandwidth and may exist only at microwave frequencies.

By combining two HP products—the HP 89410A vector signal analyzer and the HP 71910A wideband surveillance receiver—into a single measurement system, the unique capabilities of the vector signal analyzer can be used on signals with 20 MHz bandwidth at frequencies up to 26.5 GHz.

This product note describes how to configure, calibrate and operate a measurement system capable of wide bandwidth vector signal analysis.

System Description

The wideband vector signal analyzer system consists of two major components: an HP 89410A two-channel vector signal analyzer and an HP 71910A wideband surveillance receiver with a wideband IF and quadrature outputs. The vector signal analyzer provides the user interface and display, and performs all of the signal processing. The HP 71910A is basically a microwave spectrum analyzer with additional features to optimize it for surveillance and signal monitoring applications. In this application, it converts the RF or microwave signal into a baseband signal which can be further processed by the vector signal analyzer. As the front end of the measurement system, it also provides the necessary gain or attenuation.

The vector signal analyzer has two input channels, each with a bandwidth of 10 MHz. Normally, this would represent the maximum bandwidth of the signal to be analyzed. However, the HP 89410A is capable of treating the signals on each channel as two parts of the same signal. That is, the signal going into channel one represents the real part of a complex signal, and the signal going into channel two represents the imaginary part. These two signals are usually referred to as the in-phase and quadrature-phase components, or simply I and Q. The vector signal analyzer digitizes the I and Q signals which are, by themselves, real signals and then combines them internally into a single complex signal of the form I+jQ or CH1+jCH2, where j

Figure 2.
IF module generates I and Q signals using quadrature mixer. Calibrations compensate for imperfections such as DC offset, gain errors and delay mismatch. Vector signal analyzer combines I and Q signals into single, complex digital signal.
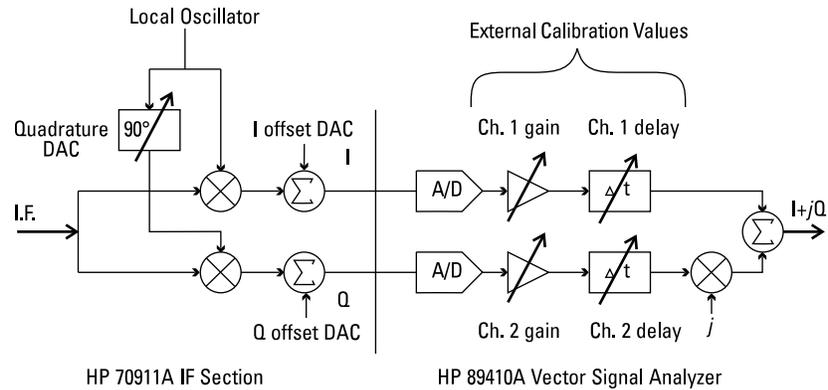
represents the square root of negative one. This new complex signal, which exists only in digital form, has a maximum bandwidth of 20 MHz, or twice the input bandwidth of the vector signal analyzer.

In the wideband vector signal analyzer system, the analog I and Q signals are generated in the wideband IF module of the microwave receiver as shown in Figure 2. The IF has a bandwidth of 100 MHz which ensures a relatively flat frequency response over the center 20 MHz used in this system. A flat IF is important in vector signal analysis. An IF with a significant amount of amplitude unflatness or group delay distortion would produce significant errors. This is especially true for modulation analysis where the IF characteristics would introduce distortion in the time domain characteristics of the signal. For example, group delay distortion in the IF would result in increased inter-symbol interference in a digitally-modulated signal. In this

system, the microwave receiver is always used at its widest bandwidth to obtain the best accuracy. The vector signal analyzer is responsible for reducing the measurement bandwidth to 20 MHz and below.

When the microwave receiver is used at frequencies below 12.8 GHz, the spectrum obtained using I+jQ is mirrored about the center frequency. There are two ways to compensate for this mirroring. The first is to simply swap the I and Q outputs. While this works, it's inconvenient and makes calibration more difficult. A simpler way to compensate for the mirroring is to conjugate the complex signal. In other words, I-jQ instead of I+jQ. The vector signal analyzer has a spectral mirror key which conjugates the CH1+jCH2 data. This corrects both time and frequency domain results without affecting the I and Q calibration.

As a measurement system, there is an obvious need for software to link the two instruments together. This

software should provide a user interface to the system, as well as provide for system calibration. An example program is available which provides these functions. The primary purpose of the example program is calibration. However, it also provides for simple control of center frequency and reference level. Once the system is calibrated and the center frequency and reference level are properly adjusted, the program is paused or terminated to provide access to all the measurement and analysis features of the vector signal analyzer. The example program is written in HP Instrument BASIC. It can be run on the HP 89410A, or on an external controller. Contact your local HP sales representative for information on how to obtain the example program.

System Configuration

This section describes the necessary components, the physical connections between components and the software required to create a wideband vector signal analysis system. Two system configurations are described: one includes the Modular Measurement System (MMS) display and one does not. If you intend to make traditional scalar spectrum measurements with the system, the display section should be included. System configuration is also simplified when the MMS display section is available. If the system will be used primarily for vector signal analysis, the MMS display is redundant since all analysis and display can be done by the HP 89410A.
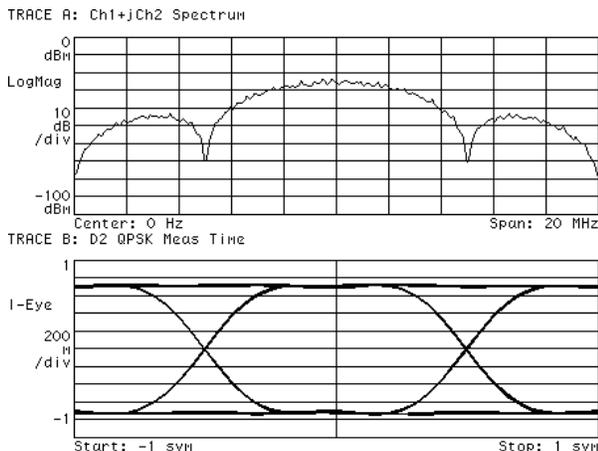

Figure 3.
The upper trace shows the spectrum of 10 Mb/s QPSK signal measured using the wideband vector signal analyzer system. The trace is 20 MHz wide. The lower trace is an eye diagram of the same signal obtained using the HP 89410A vector modulation analysis (Option AYA).

To support measurements of the complex (I+jQ) output signal of the HP 71910A, the HP 89410A vector signal analyzer must have Option AY7 (Second 10 MHz Channel) and Option AYA (Vector Modulation Analysis) installed. To control the system or perform the system calibrations via Instrument BASIC as described earlier, Option 1C2 (HP Instrument BASIC) must also be installed. The instrument firmware must be revision A.04.00 or later and can be upgraded by ordering Option UE2. Additional information on configuring the HP 89410A can be found in the HP 89400 Series Vector Signal Analyzers Configuration Guide (p/n 5964-3630E).

The HP 71910A surveillance receiver contains all the MMS components which make wide bandwidth vector signal analysis possible. Minimally, Option 004 (Analog I/Q Outputs) must be ordered and, depending on the other measurements you may want to make with the system, the MMS display (HP 70004A) may also be needed. A system without the MMS display would be ordered as an HP 71910A with Option 004, Option 011 and Option 012. In this configuration, the display is deleted, but the HP 70310A reference section is added (see Table 1).

An existing HP 71209A Option 001 spectrum analyzer can be upgraded to support wide bandwidth vector signal measurements by ordering an HP 70911A IF section (Option 098 or Option 099 firmware upgrade required). You may also need to upgrade the power supply in the HP 70001A mainframe.

The receiver firmware personality must be installed to facilitate the wideband vector signal measurements described in this paper. For additional information on firmware installation and system configuration, please refer to the HP 71910A User's Manual.

Table 1. Recommended System Configuration

HP 89410A Vector Signal Analyzer

Option AY7	Second 10 MHz Input Channel
Option AYA	Vector Modulation Analysis
Option 1C2	Instrument Basic

(Note: Must be instrument firmware revision A.04.00 or greater)

HP 71910A Wide Bandwidth Surveillance Receiver
All configurations

Option 004	Analog I/Q Outputs

(Note: Must be instrument firmware revision 94120 [B.05.00] or greater; HP 70910A receiver personality must be installed.)

Configurations without HP 70004A display

Option 011	Single mainframe configuration (deletes HP 70004A, HP 70902A, HP 70903A, HP 70310A)
Option 012	Add HP 70310A to Option 011 configuration

Other accessories

8120-1838	50 ohm BNC-BNC cable, 12" (two required)
8120-6269	50 ohm BNC-SMB cable, 24" (one required)
HP 10833A	HP-IB cable, 1 meter (one required)

A number of other modules and options are available for the family of MMS spectrum analyzers. For more information, refer to the HP 71910A Product Overview (p/n 5963-0087E) and the HP 70000 Modular Measurement System Catalog (p/n 5091-4897E).

Example configurations of the system are shown in Figure 4 (without the MMS display) and in Figure 5 (with the MMS display). The latter also includes the HP 70902A and HP 70903A IF modules which implement IF bandwidths to support traditional scalar spectrum analysis.

The rear panel views in Figures 4 and 5 show the connections between the HP 89410A and the HP 71910A. In both configurations, the 10 MHz frequency reference for the system is provided by the HP 70310A reference section. This is the recommended configuration. If your HP 89410A includes Option UFG, you have a second HP-IB connector. Referring to Figures 4 and 5, note that the HP-IB connection is made from the main HP-IB port of the HP 89410A (labeled "HP-IB") and not the connector labeled "System Interconnect". For additional information, please refer to the HP 89410A User's Guide.

Calibration Methods

The HP 89410A vector signal analyzer and the HP 71910A surveillance receiver are both capable of self calibration. To obtain the best system performance, the instruments must be calibrated together. The system calibration compensates for system-induced errors, such as unequal cable lengths for the I and Q signals. It also fine tunes factory calibrations, such as quadrature, for optimal performance. The calibration affects more than just amplitude accuracy. As shown in Figure 6, calibrations also improve the dynamic range of the system. Refer to Appendix A for a description of how errors in the signal path can affect the spectrum.

To perform the calibrations, the example software uses the 300 MHz calibrator built into the microwave spectrum analyzer. The software measures the magnitude and phase of the calibration signal while adjusting the center frequency of the microwave spectrum analyzer. It also measures the amount of residual DC in the I and Q signals. From these measurements, the software determines the IQ gain imbalance, IQ magnitude error, IQ delay mismatch, IQ quadrature error and the DC offset. It then adjusts both instruments to minimize the effects of the errors. These adjustments are depicted in Figure 2.
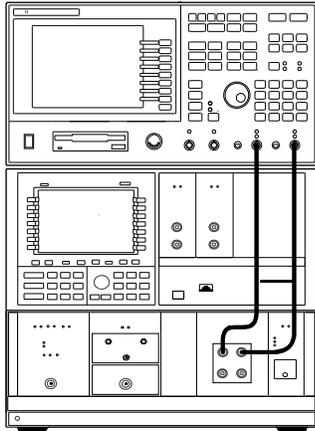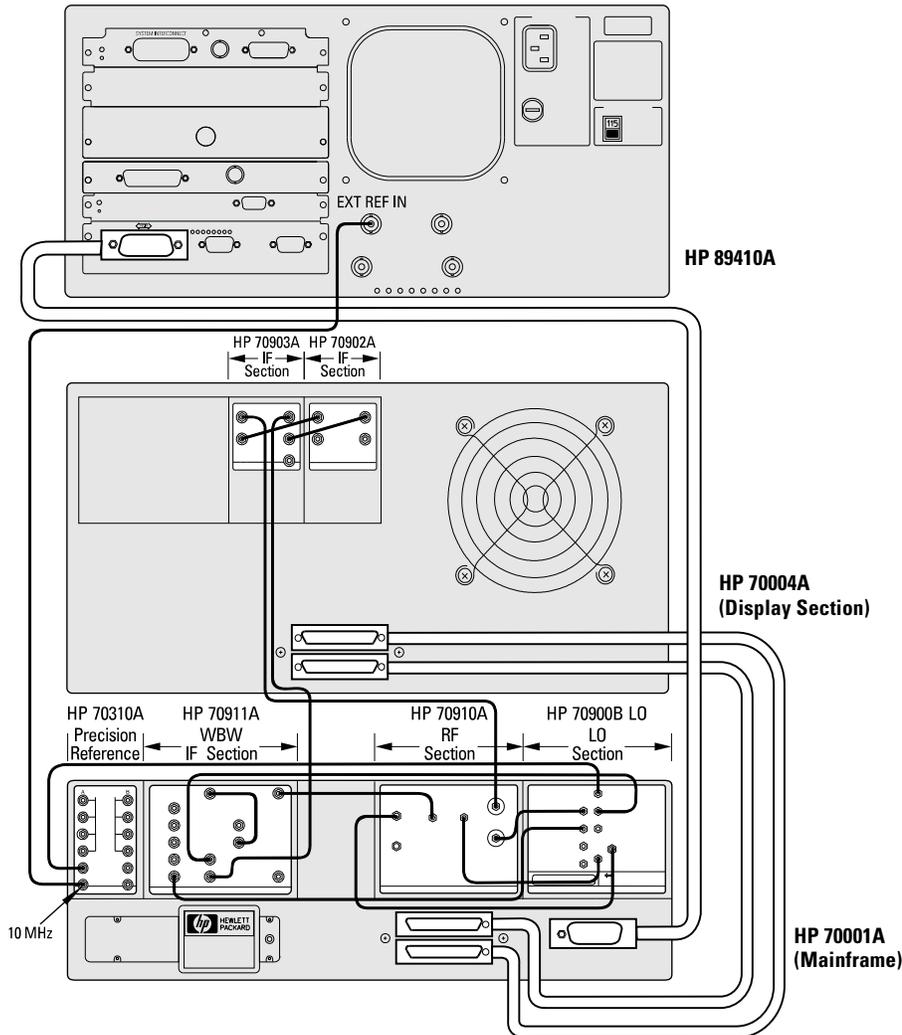
Front Panel

**Figure 4.
Front and rear panel
connections for
wideband vector
signal analyzer system
configured without
HP 70004A display.**

Rear Panel

HP 89410A

HP 71910A
Option 011

Table 2. System Connections (applies to all systems)

From		To	
70900B LO Section	LO OUT	70910A RF Section	1ST LO IN
	300 MHz OUT 1	70910A RF Section	300 MHz IN
	300 MHz OUT 2	70911A IF Section	300 MHz IN
	TUNE SPAN	70910A RF Section	TUNE SPAN
70910A RF Section	321.4 MHz OUT	70911A IF Section	321.4 MHz IN
70911A IF Section	WB VIDEO OUT	70911A IF Section	WB VIDEO IN
	321.4 MHz OUT	70911A IF Section	321.4 MHz OPT IN
	VID OUT TO LO	70900B LO Section	VIDEO IN
70310A Precision	100 MHz	70900B LO Section	100 MHz IN
Reference Section	10 MHz	HP 89410A	EXT REF IN

**Additional connections needed when the system is configured with the HP 70004A
display and the HP 70902A and HP 70903A IF Sections.**

70902A IF Section	VIDEO OUT	70903A IF Section	VIDEO IN
70903A IF Section	21.4 MHz OUT	70902A IF Section	21.4 MHz IN
	VIDEO OUT	70911A IF Section	NB VID IN
70910A RF Section	21.4 MHz OUT	70903A IF Section	21.4 MHz IN

DC Offset
Residual DC limits the dynamic range of the measurement by introducing a spurious term in the center of the spectrum (zero hertz). The DC offset calibration measures the amount of residual DC in both the I and Q channels and determines the optimal settings for the offset DACs in the HP 70911A IF module. For DC offset, the two channels are independent of one another and are calibrated separately. The following procedure describes the calibration for the I channel. The same procedure is also performed on the Q channel.

At the start of the DC offset calibration, the HP 89410A is instructed to perform an internal offset calibration. This calibration removes DC from the vector signal analyzer inputs without consideration of the input signal. This is done so that subsequent internal calibrations, which are performed automatically by the vector signal analyzer, will not affect the system calibration. After the vector signal analyzer's offset calibration, the software records the amount of DC remaining in the I channel along with the current setting of the I channel offset DAC in the HP 70911A. The DAC is then adjusted by a small amount and the amount of DC in the channel is measured again. The two DC measurements ($y1$ and $y2$) and the two DAC values ($x1$ and $x2$) are combined to determine the slope and offset of a line $y=Ax+B$ where A is the slope and B is the offset. The optimal setting for the DAC is then $-B/A$. This is the value that drives y, the residual DC, to zero.

Channel Match
Channel match is also important for preserving the dynamic range of the measurement. System errors that cause mismatch include gain imbalance, delay mismatch and frequency response differences between the I and Q signals.

A frequency response mismatch can occur in both the IF module and the

Front Panel

**Figure 5.
Front and rear panel
connections for
wideband vector
signal analyzer
system configured
with HP 70004A
display.**

Rear Panel

vector signal analyzer input channels. Only the center 20 MHz of the IF module's 100 MHz bandwidth is used (I and Q each have 50 MHz bandwidth), so the mismatch there is relatively small. The mismatches in the vector signal analyzer are compensated by the instrument's built-in calibration routines.

IQ Gain, Delay Match
The system calibration software only attempts to match gain and delay. The software determines the gain and delay parameters by measuring the I and Q signals generated when the 300 MHz calibrator is connected. To simplify the measurement, the vector signal analyzer is taken out of the channel combine mode (CH1 + jCH2) and put into two-channel mode. The center frequency of the spectrum analyzer is adjusted to 291 MHz. With a 300 MHz signal and a center frequency of 291 MHz, the I and Q outputs are both 9 MHz sine waves. In a perfect system, these 9 MHz sine waves would have a phase difference of 90 degrees and would have identical amplitudes. In a real system, the amplitudes will probably be different, and the phase difference will be something other than 90 degrees. As shown in Figure 7, the phase error has two components— error due to quadrature error and error due to IQ delay mismatch. Quadrature error in the IF module produces a phase error that is independent of the relationship between the calibrator frequency and the center frequency of the measurement. Delay mismatch produces a frequency-dependent phase error term that is zero when the center frequency and the calibrator frequency are the same (that is, the difference frequency is zero and I and Q have no AC component). To determine the delay and quadrature error, the center frequency is adjusted in 2 MHz steps from 291 MHz to 309 MHz, and the phase differences between I and Q are recorded. Then, using a least-mean-square algorithm, a line is fit to the phase error verses frequency data.

The slope of the line corresponds to the delay mismatch, and the offset to the quadrature error.

The absolute signal level and gain match are measured at the same time as the phase. At each center frequency, the magnitude squared of the signal (I^2+Q^2) is computed, as is the ratio between I and Q. The square root of the average magnitude squared is used to set the overall gain adjustment. This minimizes the absolute error across the 20 MHz bandwidth rather than the error at the center frequency. The average ratio is used to determine the gain mismatch. The gain and delay parameters are used to configure the external calibration parameters of the vector signal analyzer. It's worth noting that the errors are quite small. For example, the delay mismatch is usually less than 100 psec.

Quadrature
The final calibration is for quadrature error. Although a value for quadrature error was obtained in the channel match calibration, the number cannot be used by itself to set the quadrature adjustment DAC in the IF module as the mapping from DAC setting to phase is not known. The DAC setting for the quadrature is determined much like the DAC setting for the residual DC. First, the phase difference between the channel one and channel two signals is measured and the current DAC setting recorded. A second measurement of the phase difference is made after the DAC has been adjusted by a small amount. The two DAC values and the two phase difference values can then be used to compute how far to adjust the DAC to compensate for the quadrature error observed in the delay calibration. The example software performs this calibration with the center frequency adjusted to 309 MHz. Although the frequency is not critical, the center frequency should be different than the calibrator frequency to avoid potential problems with errors introduced by residual DC in I and Q.
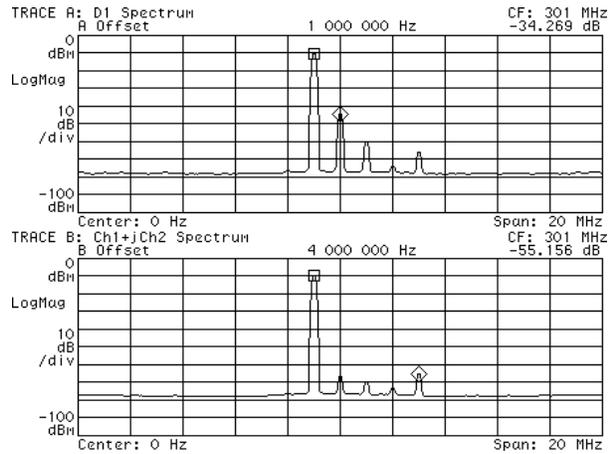

Figure 6.
Calibrations provide better accuracy and improve dynamic range. Upper trace shows spectrum of 300 MHz calibrator as measured before system calibrations. Lower trace shows measurement of same signal after system calibration.

The example software was designed to work with a 300 MHz calibrator. Any frequency can be used, provided the calibrator frequency variable in the software reflects that frequency. This might be desirable if better absolute amplitude accuracy is desired at some frequency other than 300 MHz. Other parameters, such as delay, are not greatly affected by the center frequency of the microwave spectrum analyzer.

Operation

The vector signal analyzer provides for all signal processing, display and analysis. With the exception of scalar analysis, all HP 89410A features are available in the wideband vector signal analyzer system. The system software only needs to provide control over center frequency, input attenuation, spectral mirroring and system calibration. As a convenience, it also initializes both instruments for wideband vector signal analysis. This includes, but is not limited to, changing instrument modes, setting inputs to DC coupling and adjusting RBW settings.

As previously described, the system can be configured with or without the HP 70004A display. When the display is present, the center frequency and reference level can be adjusted on the microwave receiver. While this may seem more convenient, there are good reasons for using the features found in the

example program to control the receiver from the vector signal analyzer. For example, when changing the center frequency, the example program will label the traces on the vector signal analyzer to reflect the center frequency of the measurement. This is useful since the x-axis annotation will always show a center frequency of zero hertz whenever the receiver mode is set to CH1+jCH2. Another important function that the example program handles is the setting of the mirror frequency key, used to flip or invert the spectrum of the signal about zero hertz. This operation may be necessary depending on the center frequency. In the microwave spectrum analyzer, center frequencies below 12.8 GHz will produce an output spectrum that is inverted. The frequency inversion is due to the mixing scheme used in the instrument. When setting the center frequency, the example program will check to see if the spectrum will be inverted (based on center frequency) and will set the mirror frequency key to compensate. The mirror frequency key actually causes the time data to be conjugated; that is, the sign of the Q channel data is negated. Conjugating the time data will not only mirror the spectrum, it will also correct the time data for modulation analysis.

The example program is even more useful for changing the reference level (input attenuation). When the input attenuation is adjusted in the

microwave receiver, the gain of the signal path may change. If it does, the absolute level displayed on the vector signal analyzer will be wrong. When changing the reference level, the example software will also change the gain calibration to reflect any nominal changes in the gain of the signal path. This ensures that the reference level can change without affecting the level of the result. With two instruments, there are actually two sets of input attenuators that need to be adjusted. Obviously, the input to the microwave receiver will be adjusted based on the level of the signal to be measured. The vector signal analyzer inputs are adjusted to reflect the I and Q signal levels at the output of the IF module. This setting is fixed and has been chosen as a compromise between noise and distortion performance. A variable in the example program can be adjusted to provide different levels of performance.

The system calibration described earlier is very easy to perform using the example program. Once the instruments are properly connected, the example program is started—automatically configuring the instruments for wideband vector signal analysis. Pressing the calibration softkey will initiate the calibration. Unless the example program has been modified, the program will prompt the user to connect a 300 MHz, -10 dBm signal to the input. Once connected, the calibration only

takes a few moments to perform. After calibration, the user can adjust the center frequency and reference level or exit the program to operate the vector signal analyzer. Normally, users pause the program (by pressing the BASIC key) rather than quitting the program. This allows the program to be easily continued when the user wishes to change center frequency or reference level, or to perform another calibration.

Summary

Vector signal analyzers provide measurement and analysis features not found in other types of instrumentation. These features have generally not been available for analyzing wide bandwidth signals or signals at microwave frequencies.

A vector signal analyzer system can be built which provides vector signal analysis capabilities at microwave frequencies to 26.5 GHz and with information bandwidths to 20 MHz. This system is based on the HP 89410A two-channel vector signal analyzer and the HP 71910A wideband surveillance receiver. The two channels in the vector signal analyzer are used to digitize the I and Q signals from the surveillance receiver and combine them internally into a complex digital signal of the form I+jQ.

To get the best performance from the vector signal analyzer system, the

system must be calibrated. Without calibration, IQ gain and delay imbalance, quadrature error and residual DC result in spurious signals that reduce the dynamic range of the system. No special equipment beyond the 300 MHz calibration signal built into the surveillance receiver is required for calibration.

An example program is available which performs the calibrations described in this product note. It also provides features which simplify operation of the system. These features include controlling the center frequency of the receiver, labeling the results with that center frequency, controlling the input attenuation, adjusting the vector signal analyzer to ensure correct data scaling, and adjusting the frequency mirror key to ensure proper orientation of the spectrum.

Appendix A: Complex Signals

In this section, we will use simple example signals to show how the use of complex signals allows a doubling of bandwidth relative to real-only signals and how the spectrum can be mirrored simply by conjugating the complex time data. We will also show how measurement errors can produce spurious signals which reduce the dynamic range of a measurement. A more thorough and general description of complex signals can be found in textbooks on signal processing and communications.
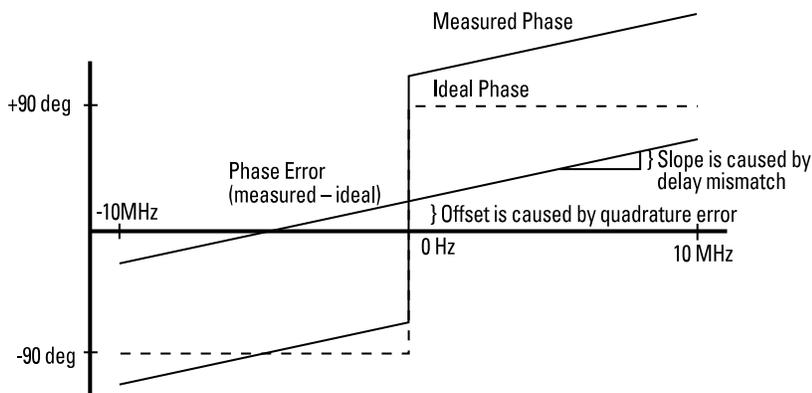
Figure 7.
For a sinusoidal input, I and Q outputs should also be sinusoidal and should have a phase difference of exactly +/-90 degrees. Quadrature error and delay mismatch between channels introduce a phase error that is a function of the relationship between the center frequency of the measurement and the frequency of the input signal.

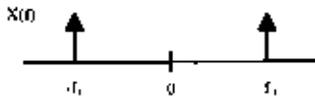Consider the following real-only sinusoidal signal:

$$x(t) = \cos(2\pi f_0 t)$$

Using the Fourier transform

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt$$

and solving for X(f), we get

$$X(f) = \tfrac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

From this result, we see that the spectrum contains two elements: a discrete spectral line at f_0 and another at $-f_0$. In general, real signals will have the property that $X(f)=X^*(-f)$. That is, the negative side of the spectrum is the complex conjugate of the positive side. This spectrum would be described as being conjugate symmetric. *Although the spectrum extends from $-f_0$ to f_0, only one side of the spectrum is unique. The usable bandwidth extends from 0 to f_0.*

If we start with the spectrum that is not conjugate symmetric and take the inverse Fourier transform, we get a complex signal. Consider two different signals with the following single-sided spectrums. We will choose these signals to represent the upper and lower frequency limits of our analysis bandwidth.
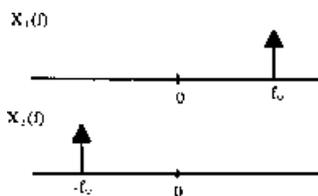
$$X_1(f) = \tfrac{1}{2}\delta(f - f_0) \text{ and } X_2(f) = \tfrac{1}{2}\delta(f + f_0)$$

Taking the inverse Fourier transform, we get:

$$x_1(t) = \tfrac{1}{2}[\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)]$$

$$x_2(t) = \tfrac{1}{2}[\cos(2\pi f_0 t) - j\sin(2\pi f_0 t)]$$

Looking at $x_1(t)$ and $x_2(t)$, we see that the individual bandwidths of the

real and imaginary components are the same as for the real-only example. In fact, $x(t)$ can be obtained by simply adding $x_1(t)$ and $x_2(t)$. Since the inverse Fourier transform of a single-sided spectrum resulted in a complex signal, it should be obvious that *a complex signal is not constrained to having a conjugate symmetric spectrum and can therefore uniquely represent signals over a bandwidth extending from $-f_0$ to f_0 which is twice the bandwidth of a real-only signal.*

Another important point in this example is that the spectrums, $X_1(f)$ and $X_2(f)$, are mirror images of each other. Looking at the time domain representation of the signals, we see that the only difference between them is that one is the complex conjugate of the other. This is a characteristic of all complex signals, not just the one used in this example. The spectrum of a complex signal can be mirrored about zero hertz by simply conjugating the time data.

In the wideband vector signal analyzer system, the microwave receiver performs a frequency translation to DC. Signals above the center frequency of the measurement appear as signals above DC in the vector signal analyzer; signals below the center frequency appear as negative frequency signals. The highest frequency I and Q signal that can be measured is 10 MHz. So, on the vector signal analyzer display, the frequency axis starts at -10 MHz and stops at 10 MHz (providing that the span is set to 20 MHz).

In a real measurement system there will be—among other impairments— a residual DC term in each channel and a gain mismatch between channels. Recall the single-sided spectrum $X_1(f)$ and its corresponding time representation:

$$x_1(t) = \tfrac{1}{2}[\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)]$$

We can modify the previous equation to include a gain error α and DC error term β on the real part of the

signal. This would represent errors in the I (channel one) portion of the signal:

$$x_1'(t) = \tfrac{1}{2}[(\alpha\cos(2\pi f_0 t) + \beta) + j\sin(2\pi f_0 t)]$$

This equation can be rewritten as:

$$x_1'(t) = \tfrac{1}{2}[\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)] + \tfrac{1}{2}[(\alpha - 1)\cos(2\pi f_0 t)] + \tfrac{\beta}{2}$$

The first component in this equation is the original signal which has a single spectral component at f_0. The second component in the equation is a real-only component which we know has spectral components at both f_0 and $-f_0$. The final component is the DC term which produces a spectral component at zero hertz. The result of the gain mismatch and the DC term is a spectrum with two extra elements as shown below. The gain error also produces a change in the level of the signal at f_0.
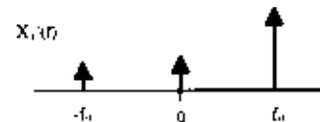
A similar approach can be used to show the effects of a quadrature error Φ on the previous signal:

$$x_1''(t) = \tfrac{1}{2}[\cos(2\pi f_0 t + \Phi) + j\sin(2\pi f_0 t)]$$

Using

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

the equation can be rewritten as:

$$x_1''(t) = \tfrac{1}{2}[\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)] + \tfrac{1}{2}[(\cos(\Phi) - 1)\cos(2\pi f_0 t) - \sin(\Phi)\sin(2\pi f_0 t)]$$

As before, the first term of the equation is the desired result. The second term is real-only with frequency components at both f_0 and $-f_0$. This shows that quadrature error can also limit the dynamic range of the measurement through the creation of spurious signals.